Formal Ontology, Propositional Identity and Truth
According to Predication
With an Application of the Theory of Types to the Logic of Modal
and Temporal Propositions

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Abstract

Standard philosophical logic tends to reduce propositions to their truth conditions. However it is quite clear that propositions with the same truth conditions are not the contents of the same attitudes and acts of thought. We can think that Paris is the capital of France without *eo ipso* thinking that Paris is the capital of France and not a real number. So we need a finer criterion of propositional identity for an adequate philosophical analysis of language, action and mind.

For that purpose, I will revise the formal ontology of the theory of types of sense and denotation of Frege and Church. I will formulate a natural logic of propositions that takes into consideration their structure of constituents and the way in which we understand their truth conditions. According to my analysis, each proposition is composed from atomic propositions (predicating each a single attribute of objects under concepts). And the truth of a proposition in any circumstance is compatible with certain possible truth conditions of its atomic propositions and incompatible with all others. So I distinguish strictly equivalent propositions whose expression requires different acts of predication or whose truth conditions are not conceived in the same way. I will explain the concise truth definition that I advocate: a proposition is true in a circumstance when its truth in that circumstance is compatible with the actual truth conditions of all its atomic propositions. I will compare my propositional logic with others: intensional and hyperintensional logics, the logic of analytic implication and that of relevance. I will also explicate a new propositional implication, *strong implication*, finer than all others, which is *a priori* known, paraconsistent, decidable and finite. Thanks to strong implication, one can characterize the minimal rationality of human agents.

Finally, I will apply my theory of types of sense and denotation to the analysis of modal and temporal propositions in the framework of the logic of ramified time. I will use the resources of proof and model theory in order to formulate my philosophical logic.
In the philosophical tradition, propositions have a double nature. On one hand, they are units of sense of a fundamental logical type. They are *senses of sentences* provided with *truth conditions*. Each proposition is *true in a circumstance* when it represents a fact which exists in that circumstance. So any analysis of the logical form of propositions requires a theory of truth. On the other hand, propositions are also the *contents of conceptual thoughts* that we, human beings, have whenever we *represent* facts of the world.\footnote{Following Descartes (1641), I distinguish conceptual thoughts from other types of thought like perception and imagination whose contents are presentations rather than representations of facts. See the Sixth Meditation.} In particular, they are the contents of *illocutionary acts* like assertions, promises and questions that we perform in the use of language. They are also the contents of our *attitudes* (beliefs, desires, intentions) towards objects and facts.

As Frege pointed out, the two intrinsic aspects of propositions are logically related. For we cannot express a propositional content in thinking and speaking without relating it *eo ipso* to the world with a certain *force*.\footnote{The notion of *force* comes from G. Frege who used the German term „Kraft“. See „Gedanke“ and „Verneinung“ Beiträge zur Philosophie der deutschen Idealismus Volume 1, 1918-19 and „Gedankengefüge“ Beiträge zur Philosophie der deutschen Idealismus Volume 3, 1923-26} Any literal meaningful utterance of an elementary sentence is always an attempt by the speaker to perform an illocutionary act with a force $F$ and a propositional content $P$.\footnote{The most common force markers are verbal mood and sentential type. For example, declarative sentences serve to make assertions. Interrogative sentences serve to ask questions and imperative sentences to give directives.} So force, sense and denotation are the three basic components of sentence meaning in the logical structure of language. And the proposition which is the sense of an elementary sentence in a context of utterance is also the content of the elementary illocutionary act that the speaker of that context would mean to perform if he or she were using that single sentence literally. The main purpose of this chapter is to formulate a *new theory of sense and denotation* that takes into account the double nature of propositions. The fact that propositions are contents of conceptual thoughts imposes to propositional logic many conditions of material and formal adequacy that logicians have unfortunately tended to neglect until now. We, human agents, have cognitive abilities which are both restricted and creative. We can only utter a finite number of sentences and make a finite number of acts of reference and predication in a context of utterance. So we can only have in mind a finite number of propositions with a finite structure of constituents. However we are free and creative. We can understand infinitely many sentences, utter new sentences and have new thoughts. We are neither omniscient nor perfectly rational. We understand most propositions without knowing whether they are true or false. We often make false assertions and sometimes believe necessarily false propositions. But we are always minimally consistent in a sense that remains to be explained. Propositional logic has to account for such facts.

In particular, the theory of truth for propositions must be compatible with the theory of success and satisfaction for illocutionary acts. By nature, illo-
Illocutionary acts have felicity conditions (Austin [1956]). Attempts to perform illocutionary acts can succeed or fail. In order to succeed to perform an illocutionary act a speaker must make a right attempt in a right context. Moreover illocutionary acts are directed at objects and facts in the world. Speakers in general attempt to achieve a success of fit between words and things. Their illocutionary acts are satisfied only if represented facts turn out to be existent. In particular, assertions are satisfied when they are true, promises are satisfied when they are kept and directives when they are obeyed. As Searle and I pointed out⁴, there is no way to elaborate an adequate theory of success and satisfaction for illocutionary acts without identifying their contents with propositions. So the formal ontology of illocutionary logic is realist. Moreover success, satisfaction and truth are logically related. So we need a unified theory of force, sense and denotation.⁵

Unfortunately, current philosophical logics of sense and denotation are incompatible with an adequate analysis of thought. Standard modal, temporal, epistemic, intensional and agentive logics are based on Carnap’s definition of the logical type of propositions. They tend to reduce propositions to their truth conditions. Such a reduction is incompatible with contemporary philosophy of language, mind and action. From a philosophical point of view, strict equivalence (the property of having the same truth values in the same circumstances) is not a sufficient criterion of propositional identity. Many speech acts with the same illocutionary force and strictly equivalent propositional contents have different success and sincerity conditions. Thus the assertion (or belief) that Ottawa is the capital of Canada is different from the assertion (or belief) that Ottawa is the capital of Canada and not a real number.

I will formulate an analysis of the logical form of propositions which is adequate for the purpose of the theory of speaker and sentence meaning. On my view, the primary units of meaning in the use and comprehension of language are not isolated propositions but complete illocutionary acts of the form F(P) provided with felicity rather than truth conditions. So no proposition can be the sense of a sentence in a context of utterance according to a possible semantic interpretation unless it is also a possible content of illocutionary act. My notion of proposition is general; it covers senses of all types of sentence (declarative or not). The same proposition can be the common sense of sentences of different syntactic types e.g. “You will help Paul” and “Please, help Paul!” just as it can be the common content of illocutionary acts with different forces.⁶

This chapter has the following content. The first section presents the formal

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⁵In order to be satisfied, an elementary illocutionary act must have a true propositional content. So the traditional correspondence theory of truth for propositions is part of the more general theory of satisfaction for illocutionary acts.

⁶Non declarative sentences are then also logically related by virtue of the truth conditions of propositions which are their senses. For example, the sentences “Do it!” and “You could not do it” express illocutionary acts which are not simultaneously satisfiable.
ontology of my theory of types of sense and denotation and the second section my analysis of the logical type of proposition. The third section formulates a concise definition of truth according to predication and explicates a new relation of strong propositional implication which is important for the account of rationality. The fourth section illustrates my theory of sense; it proceeds to the analysis of modal and temporal propositions within the logic of ramified time. That section defines the ideographic object language of a rich philosophical logic. The fifth section presents the formal semantics of that logic and the sixth section an axiomatic system where valid laws are provable. Finally, the last section states a series of important valid laws governing truth, propositional identity and strong implication. It shows striking differences existing between my logic of sense and current modal and intensional logics of Lewis, Carnap and Montague, Hintikka’s epistemic logic, Belnap’s logic of relevance, Parry’s logic of analytic implication and Cresswell’s hyperintensional logic.

1 THEORY OF TYPES

On the basis of preceding considerations about thought and meaning, I advocate a Frege – Church formal ontology much richer than that of Russell. I propose to stratify as follows the universe of discourse in the theory of types of philosophical logic:

1. **There are three primitive types of denotation**: the type $e$ of individuals, the type $t$ of truth values and the type $s$ of success values. Individuals are particular objects like material bodies and persons existing in actual or possible courses of the world. They are objects of reference of the simplest logical kind. There is at least one individual in the world. So there is a non empty set of individuals. The two truth values are truth and falsity and the two success values success and insuccess.

2. **There are two primitive types of sense**: the type $c$ of concepts of individuals and the type $r$ of attributes of individuals. Properties of individuals like being alive and, for each number $n \geq 2$, relations of degree $n$ between individuals like being taller than are attributes of individuals. In my symbolism, Concepts is the set of individual concepts and Attributes the set of attributes of individuals. So the set of primitive senses is the union of Concepts and Attributes.

As Frege and Church pointed out, there is a fundamental relation of correspondence between senses and denotations in the universe of discourse. Actual

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7For a clear presentation of the differences between the two ontologies see David Kaplan “How to Russell a Frege-Church” *The Journal of Philosophy* 716-729, 1971

8The theory of types of sense and denotation underlying my stratification comes from Alonzo Church “A Formulation of the Logic of Sense and Denotation” in P. Henle *et al* (eds) *Structure, Method and Meaning* Liberal Arts Press 1951

9The law of excluded middle holds for success and insuccess just as it holds for truth and falsity. Either an illocutionary act is performed or it is not performed in a speech situation. Failure is a special case of insuccess which occurs only when a speaker makes an unsuccessful attempt to perform the illocutionary act.
denotations of certain types correspond to senses in possible circumstances.\footnote{The notion of \textit{circumstance} comes from D. Kaplan (1979) “On the Logic of Demonstratives” propositions are true in possible circumstances. a possible circumstance can be a moment of time, a possible world, a pair of a moment of time and history. All depends on the logic under consideration.}

Thus \textit{propositions}, which are \textit{senses of sentences}, have \textit{truth values} as denotations; they are either true or false in each circumstance. Concepts of individual objects, which are senses of referring expressions like the king of France, have single individuals as denotations: they apply to at most one individual object in each circumstance. The denotation corresponding to a sense can of course vary from one circumstance to another. In the logic of ramified time and of action, each \textit{circumstance} is a pair $m/h$ of a moment of time $m$ and of a history $h$ to which that moment belongs. Different persons have been king of France in the past. In certain circumstances no individual falls under a concept. There is no actual king of France at the present moment. There is a next king of France in a circumstance $m/h$ when someone becomes king at a moment $m'$ posterior to $m$ in the history $h$.

Properties of individual objects, which are senses of unary predicates, have sets of individuals under concepts as denotations: a certain number of \textit{individuals under concepts} possess them in each circumstance. Relations of degree $n$ between individuals are senses of $n$-ary predicates where $n \geq 2$. In thinking and speaking, we predicate relations of degree $n$ in a certain order. We apply them successively to $n$ objects of reference. The order of predication is expressed in language use by the syntactical order in which referential expressions occur in atomic clauses. In thinking that Goliath is taller than David we predicate the relation of being taller first to David next to Goliath. Most binary relations between individuals are not symmetric. So these relations are satisfied by individuals under concepts in a certain order. Goliath is taller than David but David is not taller than Goliath. This is why attributes are said to be satisfied by sequences in the calculus of predicates.\footnote{Properties are attributes of degree 1; they are satisfied by sets of (unary sequences of) individuals under concepts.}

As Frege [1892] pointed out, the truth value of many propositions depends on the sense rather than the denotation of their propositional constituents. Certain attributes of individuals are \textit{intensional}: they are satisfied by sequences of individuals under some concepts without being satisfied by the same sequences of individuals under other concepts. Thus the relation of desire is intensional: Oedipus desired to marry Jocasta, the Queen of Thebes; but he did not desire to marry his mother. For that reason, denotations of attributes are sequences of individual concepts (or of individuals under concepts) rather than sequences of pure individuals in philosophical logic.\footnote{See D. Lewis (1972) “General Semantics.”} There are however \textit{extensional attributes} whose denotation only depends on the denotations of the concepts which satisfy them. So are the property of being tall and the relation of moving something. If Jocasta, the queen of Thebes is tall so is Oedipus’ mother. \textit{Extensional attributes} are satisfied by a sequence of individuals under concepts
in a circumstance when they are satisfied by the sequence of individuals that fall under these concepts in that circumstance. For the sake of simplicity, I will often give examples of extensional attributes.

3. **Each type is a subtype of more general types.** For any pair of types $\alpha$ and $\beta$ of entities of the universe of discourse, there is the derived type $\alpha \cup \beta$ of all entities which are of the type $\alpha$ or $\beta$. By definition, the set $U_{\alpha \cup \beta}$ of entities of type $\alpha \cup \beta$ is the union $U_\alpha \cup U_\beta$ of the set $U_\alpha$ of entities of type $\alpha$ and of the set $U_\beta$ of entities of type $\beta$. Thus concepts and attributes have the more general type of *propositional constituents*.\(^{13}\) For example, $c \cup r$ is the type of propositional constituents of first order propositions about individual objects.\(^{14}\)

As in intensional logic, the set of types of entities is closed under the following two other operations:

4. For any pair of types $\alpha$ and $\beta$, **there is the derived type $(\alpha \beta)$ of functions from the set of all entities of type $\alpha$ into the set of all entities of type $\beta$.** By definition, the set $U_{\alpha \beta}$ of entities of type $(\alpha \beta)$ is the set of functions $U_\alpha \rightarrow U_\beta$. Thus $(tt)$ is the type of unary truth functions and $t(tt)$ that of binary truth functions. $(et)$ is the type of (characteristic function of) sets of individuals and $e(et)$ that of sets of pairs of individuals. As usual, sets of $n$-ary sequences of entities of the type $\alpha$ are of the type $(\alpha(...(\alpha)\ldots))$ with $n$ left and $n$ right parentheses.

5. Finally, for any type $\alpha$ of entities, **there is the derived type $\#\alpha$ of intensions whose extensions are entities of type $\alpha$.** An intension\(^ {15}\) of type $\#\alpha$ is a function from the set $\text{Circumstances}$ of all possible circumstances into the set of entities of type $\alpha$. Thus the set $U_{\#\alpha}$ of entities of type $\#\alpha$ is the set of functions $\text{Circumstances} \rightarrow U_\alpha$. For example, Carnapian truth conditions are intensions of type $\#t$: they are functions which associate with any possible circumstance a single truth value.

All types of first order senses and denotations of the universe of discourse can be obtained from the few primitive types named above by applying the three operations on types that I have defined. From Carnap we know that **each sense to which correspond entities of type $\alpha$ has a characteristic intension of type $\#\alpha$,** namely the function which associates with any possible circumstance the actual entity which is the denotation of that sense in that very circumstance. So any proposition has its characteristic Carnapian truth conditions which associate with each possible circumstance the true if and only if that proposition is true in that circumstance. Unfortunately traditional intensional logic has tended to identify senses with their characteristic intension. So propositions are reduced to truth conditions: their type $p$ is $\#t$ in the modal logic of Carnap, Prior, Montague, Kaplan, Kripke, Belnap and most other logicians. *Strictly equivalent* propositions, which are true in the same possible circumstances, are then identified. There is only one necessarily true proposition as

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\(^{13}\) The present theory of types is cumulative. Unlike Russell I admit the type of sets whose elements are of different inferior types.

\(^{14}\) I will only consider first order propositions in the present chapter.

\(^{15}\) The term and notion of intension come from Carnap (1956) *Meaning and Necessity*, University of Chicago Press.
well as only one necessarily false proposition according to current philosophical
logic.

However, it is clear that most strictly equivalent propositions do not have the
same cognitive values. In particular, they are not substitutable *salva felicitate*
within the scope of illocutionary forces and psychological modes. For example,
one can assert (and believe) that a big city is a city without asserting (or believ-
ing) *eo ipso* that $\sqrt{2}$ is an irrational number, even if these two assertions (and
beliefs) are both true in all possible circumstances. Philosophies of language
and mind require a much finer logic of sense. Just as the same denotation can
correspond to different senses, the same intension can be common to different
senses in the deep logical structure of language.

2 THE LOGICAL FORM OF PROPOSITIONS

In order to seriously take into account the fact that propositions are always
expressed in the attempted performance of illocutionary acts, I have advocated
in *Meaning and Speech Acts* and other papers a *natural predicative logic
of propositions*. My main idea was to explicate the logical type of proposition
by mainly taking into consideration the acts of predication that we make in
expressing and understanding propositions. My propositional logic according to
predication is based on the following principles:

2.1 Propositional constituents are senses and not pure de-
notations.

As Frege (1892) pointed out, we cannot refer to objects without subsuming
them under senses and without predicating of them attributes. Thus referential
and predicative expressions of sentences have a sense in addition to a possible
denotation in each possible context of utterance. When we speak literally, we
conceive the concepts and attributes which are the senses of the referential and
predicative expressions that we use. Moreover we refer to the objects which
fall under these concepts in the context of utterance. Frege’s argument against
direct reference is valid if propositions are contents of thought. Otherwise, we
would be totally inconsistent. We can make mistakes and assert, for example,
that Hesperus is not Phosphorus. But we never intend to make the absurd
assertion that Hesperus is not Hesperus. Consequently, there are no *singular
propositions* whose constituents are pure individual objects in my formal ontol-
ogy, contrary to what Russell and others advocate in defending direct reference
or externalism. All our objects of reference are *objects under concepts*.

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and Formal Semantics” and D. Vanderveken (2001) “Universal Grammar and Speech Act
Theory”.

17This example comes from a talk by D. Kaplan at McGill University. “Hesperus” and
“Phosphorus” are different proper names of Venus.
2.2 Propositions have a structure of constituents.

In speaking and thinking conceptually, we always predicate in a certain order attributes of our objects of reference. Just as any clause of an elementary sentence is composed from one or several atomic clauses where predicates of degree n are syntactically combined in a certain order with a number n of complete referential expressions, each propositional content that we have in mind is composed from one or several atomic propositions whose expression consists in an act of predication. To express an atomic proposition is just to predicate in a certain order an attribute of degree n of n individual objects under concepts. So each atomic proposition has a number n+1 of propositional constituents. It contains a main attribute of individuals $R_n$ of degree n and n individual concepts $u_1^c, \ldots, u_n^c$. And it is true in a circumstance when the sequence corresponding to the order of predication of its n individuals under concepts belongs to the actual denotation of its attribute in that circumstance. The order of predication is important only when it changes the truth conditions. The relation of identity is by nature symmetric. So the order in which we predicate the relation of identity of two entities is not important. We express the same atomic proposition in thinking that the morning star is the evening star and in thinking that the evening star is the morning star. On the contrary, the relation of admiration is not symmetric. A person can admire someone without being admired by him or her. So we express different atomic propositions when we predicate in a different order the relation of admiration of two objects of reference. We do not think that Napoleon admires Chirac when we think that Chirac admires Napoleon. Atomic propositions are identical when they have the same propositional constituents and they are true in the same possible circumstances. So atomic propositions are of the type $a = ((e \cup r)t)((\#t)t)$. Each atomic proposition $u_a$ is a pair whose first term $id_1(u_a)$ is a finite non empty set of propositional constituents and whose second term $id_2(u_a)$ is a set of possible circumstances. And the set $U_a$ of atomic propositions is a special proper subset of the set $(P(Attributes) \cup P(Concepts)) \times P(Circumstances)$ in my logic of sense.

Elementary propositions like the proposition that the actual pope speaks Russian are composed from a single atomic proposition. Complex propositions like the proposition that the actual pope is Polish and speaks Russian are composed from several atomic propositions. I will say that two propositions have the same structure of constituents when they are composed from the same atomic propositions. In order to be identical two propositions must have the same structure of constituents.

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18To predicate is not to judge; it is just to apply an attribute to arguments. So acts of predication are purely propositional; they are independent from force. One can predicate a property of an object of reference in asking a question as well as in making an assertion.
2.3 An adequate explication of truth conditions must take into account the effective way in which we understand such conditions.

To understand the truth conditions of an elementary proposition is not to know the actual truth value of its single atomic proposition in each possible circumstance. We understand the elementary proposition that the biggest whale is a fish without knowing *eo ipso* that it is necessarily false. We discovered in the course of history that whales are mammals. We understand expressed elementary propositions without knowing whether they are true or not in the context of utterance. Consider elementary propositions of the simplest kind which predicate an extensional property of an individual under concept. In understanding such elementary propositions we just understand that they are true in all (and only) the circumstances where the individual which falls under their concept possesses their characteristic property. We in general do not know by virtue of competence actual denotations of propositional constituents in the context of utterance. We often refer to an object under a concept without being able to identify the object falling under that concept. Someone who says that Julie’s murderer is wounded can just refer to whoever in the world is her murderer. Our knowledge of the world is not only partial. Some of our beliefs are false. We can wrongly believe that objects of reference possess a certain property. So we can refer to objects which do not fall under the concept that we have in mind.19 Furthermore, the objects to which we refer can possess predicated properties in many different ways. Julie’s murderer could be wounded in various places.

From a cognitive point of view, different possible denotations could then correspond according to us to an expressed attribute or concept in each circumstance. In apprehending propositional constituents we rarely identify their actual denotations. We just presuppose that they have one in each circumstance. However we are able to consider by virtue of competence possible denotations that these senses could have in each circumstance. We might ignore who is Julie’s murderer and not be sure that her murderer is wounded at a moment of utterance. But we can at least think of various people who could have murdered Julie and who could be wounded at that moment.

Any speaker who conceives propositional constituents can in principle assign to them possible denotations of the appropriate type in the circumstances that he or she considers. Our possible assignments of a denotation to propositional constituents associate a single individual (or no individual at all) with each concept and a unique set of individuals under concepts with each property in each possible circumstance. Let us give a few examples. According to a first possible denotation assignment, my friend Paul would be Julie’s murderer and he would also be wounded at the moment $m$ according to history $h$. According to a second one, the chief of police, Julius, would be Julie’s murderer but he would not be wounded at all in the circumstance $m/h$. According to a third

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19See Kripke [1977] “Speaker’s Reference and Semantic Reference”
one, nobody would be Julie’s murderer but Paul. Julius and other people would be wounded in \( m/h \), and so on. We clearly do not know a priori which possible assignments of a denotation to such senses match the reality. But we are at least capable by virtue of competence of distinguishing denotation assignments according to which the atomic proposition that Julie’s murderer is insane is true from denotation assignments according to which it is false. Thus we know that the atomic proposition is true in the circumstance \( m/h \) according to the first possible assignment considered above and that it is false in that circumstance according to the two others.

Most atomic propositions have a lot of possible truth conditions in our interpretations: they could be true according to us in a lot of different sets of possible circumstances given the various denotations that their propositional constituents could have in circumstances. From a logical point of view, there are as many possible truth conditions for an atomic proposition as there are different sets of possible circumstances where that proposition could be true. Possible truth conditions are of the type \( \#t \). An atomic proposition could be true in a possible circumstance according to an agent when it is true in that circumstance according to at least one way that agent could assign a denotation to its propositional constituents. Any agent who takes into consideration a number \( n \) of different possible circumstances can in principle assign \( 2^n \) different possible truth conditions to atomic propositions in his or her interpretation. Among all possible truth conditions for an atomic proposition \( u_a \), there are of course its actual Carnapian truth conditions to which correspond the set of possible circumstances \( id_2(u_a) \) where it is true.

Many atomic propositions are logically related. If Paul is Julie’s husband and Julie’s husband is taller than Jim, then Paul is taller than Jim. The first two atomic propositions could not be true in a circumstance unless the third one is also true in that very circumstance. Possible truth conditions of atomic propositions are logically related. So our possible interpretations respect meaning postulates of a logical nature in assigning possible denotations to propositional constituents in circumstances. Our possible valuations of senses assign to atomic propositions possible truth conditions that they all could have together.

From a logical point of view, possible valuations of senses are functions assigning to concepts and attributes denotations of the appropriate type in possible circumstances. By definition, \( val(u_c) m/h \in Individuals \) when an individual object falls under the individual concept \( u_c \) according to the possible valuation \( val \). Otherwise, \( val(u_c) m/h = \emptyset \).\(^{20}\) And \( val(R_n)(m/h) \in P(Concepts^n) \) for any attribute \( R_n \) of degree \( n \). So the set \( Val_{u/c}r \) of possible valuations of propositional constituents is a proper subset of the set \( (Concepts \rightarrow (Circumstances \rightarrow \)
Among all possible valuations of propositional constituents there is of course the real valuation (in symbols $\text{val}^*$) which associates with concepts and attributes their actual denotation in each possible circumstance. Thus $\text{val}^*(u_c)m/h$ is the object which really falls under individual concept $u_c$ (in case there is one) and $\text{val}^*(R_n)m/h$ the sequence of objects under concepts which really satisfy attribute $R_n$ in the circumstance $m/h$. We, human agents, are not aware of actual denotations of most senses in most circumstances. So our possible valuations of senses can associate a non actual denotation to many propositional constituents in many circumstances. A possible valuation $\text{val} \in \text{Val}$ is a real valuation of an individual concept $u_c$ and of an attribute $R_n$ when, for any circumstance $m/h$, $\text{val}(u_c) = \text{val}^*(u_c)m,h$ and $\text{val}(R_n)m/h = \text{val}^*(R_n)m/h$.

However all possible valuations of senses respect meaning postulates for logical attributes and operations on attributes that are imposed by their logical form. We all know by virtue of competence that individuals that we subsume under different concepts are identical in a circumstance when these concepts apply to the same individual in that circumstance. So all possible valuations $\text{val}$ assign the same denotation to the identity relation $=$ between individuals which is a logical universal. According to any interpretation a pair of individuals under concepts $u^1_c,u^2_c \in \text{val}(=)m/h$ if and only if $\text{val}(u_1^1)m,h = \text{val}(u_2^2)m,h$. Individual concepts have internal properties. So any object of reference which falls according to a valuation under certain concepts (e.g. the concept of being Oedipus’ mother) in a circumstance possesses according to that valuation essential properties (e.g. to be a woman) in all circumstances where it is existent. So certain atomic propositions (e.g. that Oedipus’ mother is male) are contradictory: they are false in all circumstances according to every possible valuation of an interpretation. Furthermore, objects of reference which possess certain properties (e.g. to be fallible) in a circumstance according to certain valuations must have other properties (e.g. to make a mistake) in the same or other possible circumstances.

As one can expect, each possible valuation of propositional constituents associates particular possible truth conditions with all atomic propositions containing these constituents. By virtue of its logical form, an atomic proposition $R_n(u^1_c,\ldots,u^n_c)$ predicating an attribute $R_n$ of $n$ individuals under concepts $u^1_c,\ldots,u^n_c$ in that order is true according to a valuation $\text{val}$ in a circumstance $m/h$ if and only if $\langle u^1_c,\ldots,u^n_c \rangle \in \text{val}(R_n)m/h$.\footnote{Of course, when the attribute $R_n$ is extensional, atomic proposition $R_n(u^1_c,\ldots,u^n_c)$ is true according to $\text{val}$ at $m/h$ if and only if $\langle \text{val}(u^1_c)m,h,\ldots,\text{val}(u^n_c)m,h \rangle \in \text{val}(R_n)m/h$.} So to each possible valuation of propositional constituents corresponds a unique possible valuation associating with all atomic propositions possible truth conditions that they could all have together. Possible valuations $\text{val}$ of atomic propositions are of the type $\text{a} \# \text{t}$. They belong to a proper subset $\text{Val}_n$ of the set of functions $U_n \to \mathcal{P} \text{ Circumstances}$. So any possible valuation of atomic propositions $\text{val}+ \in \text{Val}_n$ is the extension of...
a possible valuation of senses $val \in Val_{cl, r}$ such that $m/h \in val^+ (R_n((u^1_1, \ldots, u^n_n))$ if and only if $< u^1_1, \ldots, u^n_n > \in val^+ (R_n) m/h$). Consequently, the distinguished real valuation of senses $val^*$, which assigns to all attributes and concepts their actual denotation in each circumstance, determines a real valuation $val^*+$ of atomic propositions that associates with each of them its actual Carnapian truth conditions in the reality.

By definition, an atomic proposition $u_a$ is true at a moment $m$ according to a history $h$ if and only if $m/h \in val^*+(u_a)$. For $val^*+(u_a)$ is by definition $id_2(u_a)$.

We a priori know actual truth values of few propositions, just as we a priori know the actual denotations of few propositional constituents. Firstly, there are few tautological (or contradictory) atomic propositions that we a priori know to be necessarily true (or false) by virtue of competence. In my terminology, tautological atomic propositions predicate of a sequence of objects of reference an attribute that we a priori know that they satisfy (e.g. that Platon’s mother is a woman) given the logical form of these concepts and of that attribute.

Their only possible truth condition in our mind is then the set of all possible circumstances. On the contrary, contradictory atomic propositions predicate of a sequence of objects of reference an attribute that we a priori know that they do not satisfy (e.g. that Platon is different from himself). Their only possible truth condition in our interpretations is the empty set of all possible circumstances. Elementary propositions containing a tautological (or contradictory) atomic proposition are exceptions. They are necessarily and analytically true (and false). Most elementary propositions are contingently, a posteriori and synthetically true or false. We need to observe the world in order to know whether they are true or false.

Moreover, the truth of most complex propositions containing several atomic propositions is compatible with various possible ways in which objects could be. Think of disjunctions, past and future propositions, historic possibilities, etc. Consider the past proposition that the pope was sick. In order that it be true in a given circumstance, it is sufficient that the pope be sick in at least one previous circumstance. So the truth of that past proposition in any circumstance $c$ is compatible with a lot of possible truth conditions of the atomic proposition attributing to the pope the property of being sick.\footnote{It is compatible with all the possible truth conditions of that atomic proposition according to which it is true in at least one anterior circumstance.}

When a proposition contains several atomic propositions, its truth in a circumstance is compatible with certain possible valuations of its atomic propositions and incompatible with all others.\footnote{Possible truth conditions of other atomic propositions do not matter. The truth of any proposition is compatible with all their possible truth conditions.} We can ignore actual truth conditions. But we always distinguish, when we have a propositional content in mind, the possible valuations of its atomic propositions which are compatible with its truth in a possible circumstance, from those which are not. In making such a distinction our mind draws a kind of truth table. Thus we know that the truth of an elementary proposition in any circumstance is compatible with all and only the possible valuations of its single atomic proposition according to which it is true

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in that very circumstance. We know that the truth of a propositional negation
\( \neg P \) in a circumstance is compatible with all and only the possible valuations
of its atomic propositions which are incompatible with the truth of \( P \) in that
circumstance. And that the truth of the modal proposition that it is universally
necessary that \( P \) is compatible with all and only the possible valuations
of its atomic propositions which are compatible with the truth of \( P \) in every
possible circumstance. So the type of truth conditions of complete propositions
is \( \#((a\#t)t) \) in my logic of sense. \( U_{\#((a\#t)t)} = (\text{Circumstances} \rightarrow P \ (U_a \rightarrow P \ \text{Circumstances}) \).

As Wittgenstein pointed out in the \textit{Tractatus}, there are two limit cases of
truth conditions. Sometimes the truth of a proposition is compatible with all
possible ways in which objects could be. It is a tautology. Sometimes it is
incompatible with all of them. It is a contradiction. In my approach, a \textit{tautology}
is a proposition whose truth in any circumstance is compatible with all
the possible truth conditions of its atomic propositions. And a \textit{contradiction} a
proposition whose truth is compatible with none. By virtue of their logical form,
tautologies are then true according to all possible valuations of atomic pro-
ositions and contradictions according to none. Thus tautologies are a particular
case of necessarily true propositions just as contradictions are a particular case
of necessarily false propositions. Unlike what is the case for other necessarily
true or necessarily false propositions, we \textit{a priori} know that tautologies are neces-
sarily true and that contradictions are necessarily false when we apprehend
their logical form. Tautologyhood and contradiction are epistemic as well as
logical and metaphysical notions.

### 2.4 The new criterion of propositional identity. Identical
propositions have the same structure of constituents
and their truth in each circumstance is compatible
with the same possible truth conditions of their atomic
propositions.

The type \( p \) of propositions is \( (at)((a\#t)t)t \). Thus the set \( U_p \) of propositions
is included in the set \( P \ U_a \times (\text{Circumstances} \rightarrow P \ (U_a \rightarrow P \ \text{Circumstances}) \).
From a logical point of view, each proposition \( P \) has a characteristic finite set of
atomic propositions (in symbols \( \text{id}_1 P \)) and a characteristic intension (in symbols
\( \text{id}_2 P \)) which associates with any possible circumstance the set of possible valua-
tions of its atomic propositions which are compatible with its truth in that very
circumstance. My criterion of propositional identity is stronger than that of
modal, temporal, intensional and relevance logics. Strictly equivalent propositions
composed out of different atomic propositions are no longer identified. We
do not make the same predications in expressing them. So the propositions that
the morning star is the morning star and that the evening star is the evening
star are different tautologies. Their propositional constituents are different.

Furthermore, unlike Parry\textsuperscript{24} I do not identify all strictly equivalent proposi-

\textsuperscript{24}W.T. Parry (1933) “Ein Axiomsystem fuer eine neue Art von Implikation (analytische
tions with the same structure of constituents. Consider the elementary proposition that the biggest whale is a fish and the contradiction that the biggest whale is and is not a fish. They are both necessarily false and contain the same single atomic proposition. But they do not have the same cognitive value. We can believe that whales are fishes. But we could not believe that whales are and are not fishes. In my logic, such propositions are different because their truth is not compatible with the same possible truth conditions of their atomic proposition.\textsuperscript{25} However, as we will see later, my criterion of propositional identity is less rigid than that of intensional isomorphism in Cresswell’s hyperintensional logic\textsuperscript{26}. For all Boolean laws of idempotence, commutativity, distributivity and associativity of truth functions remain valid laws of propositional identity.

2.5 The set of propositions is recursive.

*Elementary propositions are the simplest propositions.* They contain a single atomic proposition and are true in all circumstances where that atomic proposition is true.\textsuperscript{27} All other propositions are more complex: they are obtained by applying to simpler propositions operations which change their structure of constituents or truth conditions. Truth functions are the simplest propositional operations. Complex propositions composed by truth functions have all and only the atomic propositions of their arguments. And their truth value in a circumstance only depends on the truth values of their arguments in that very circumstance. Thus the conjunction \( P \land Q \) and the disjunction \( P \lor Q \) of two propositions have the atomic propositions of their arguments \( P \) and \( Q \). They only differ by their truth conditions.\textsuperscript{28}

Unlike truth functions, quantification, modal and temporal operations on propositions change the structure of constituents as well as truth conditions. When we think that all objects are such that God has knowledge of them, we predicate of Him the property of omniscience, namely that He knows everything. Such a generalized proposition contains a new atomic proposition predicating a generalization of the attribute of its argument. It is moreover true in a circumstance when that new atomic proposition is true in that circumstance.\textsuperscript{29}

\textsuperscript{25}On the one hand, a lot of possible truth conditions are compatible with the truth in any circumstance of the elementary proposition that the biggest whale is a fish, namely all those according to which its atomic proposition is true in that circumstance. So we can believe it. On the other hand, the truth of the contradiction is not compatible with any possible truth condition of its atomic proposition. Since we know this a priori in understanding its truth condition, we cannot believe it.

\textsuperscript{26}Max Cresswell (1975) “Hyperintensional Logic”

\textsuperscript{27}Thus the set of atomic propositions of any elementary proposition \( P \) is a singleton \( \{ u_a \} \) and its intension \( u_{(a \# t)} \), is such that, for any circumstance \( c \), \( u_{(a \# t)}(m/h) = \{ f \in (U_a \rightarrow P \text{ Circonstances}) / m/h \in f(u_a) \} \).

\textsuperscript{28}As is well known, the truth of the disjunction in a circumstance is compatible with all the possible valuations of its atomic propositions which are compatible with the truth of at least one of its arguments in that circumstance. But the truth of the conjunction is only compatible with the possible valuations which are compatible with the truth of both arguments.

\textsuperscript{29}As I [1997] pointed out in “Quantification and the Logic of Generalized Propositions”
Modal and temporal operations also change the structure of constituents. They add new atomic propositions. When we think that it is universally necessary that God does not make mistakes, we do more than attribute to God the property of not making mistakes. We also attribute to Him the modal property of infallibility, namely that He does not make mistakes in any possible circumstance. The property of infallibility is the necessitation of the property of not making mistakes. Modal propositions according to which it is necessary that things are such and such contain new atomic propositions predicing the necessitation of attributes of their argument. Unlike quantification, modal and temporal operations are however intensional: the truth value of a modal or temporal proposition in a circumstance depends on the truth values of its argument in other possible circumstances. Thus the truth in a circumstance of the temporal proposition that it has always been the case that P is compatible with all possible valuations of its atomic propositions which are compatible with the truth of P in all anterior circumstances.

3 TRUTH ACCORDING TO PREDICATION

Thanks to the new explication of the logical type of proposition, my logic of sense and denotation offers a new concise definition of truth by correspondence and articulates better the logical structure of propositions. In the philosophical tradition, from Aristotle to Tarski, truth is based on correspondence with reality: true propositions correspond to existing facts. Objects of reference have properties and stand in relations in actual and possible circumstances. Atomic propositions have therefore a well determined truth value in each circumstance depending on the actual denotation of their attributes and concepts and the order of predication.

Moments of time represent possible complete states of the actual world at an instant in the logic of ramified time and action. The past is unique but the future is open. We, human agents, live in an indeterminist world. So various alternative incompatible moments could directly follow a moment. Actual moments of time some represent actual complete states of the world. The present moment, which represents “all nature now” (as Whitehead says), is actual as well as all moments which are anterior to it. These moments belong to the actual course of history of this world. Atomic propositions which are true at a moment (or time interval) in the actual course of history of the world represent facts (states of affairs, events or actions) which exist or happen at some instant (or time interval) in the actual world. As Wittgenstein noticed at the beginning of the Tractatus, “The world divides into facts” (1.2) not into objects. Because of

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second order predication is not needed for the logical analysis of generalization over individual objects. One can remain in the simpler formal ontology of first order attributes. Using the classical logic of attributes one can analyze new predicated attributes as being generalizations of first order attributes. The property of omniscience is a universal generalization of the relation of knowing in the intensional logic of attributes. See G. Bealer Quality and Concept [1982] for an explanation of the operations of generalization on attributes.

\[30\]We are free because we could do something else from what we actually do.
indeterminism, there are a lot of possible historic continuations of the present moment. We ignore which of them will turn to be actual. We can just assume that there is a single one (if the world continues).

Many possible circumstances that we consider belong to a possible non actual course of history of the world. Such non actual circumstances are however possible. They belong to what I will call the logical space of reality. Propositions which are true in possible circumstances represent possible facts that would exist if the possible course of history to which they belong were actual. The world is part of reality. So in order to describe the real world and represent existing facts (for example, that certain objects are soluble) in actual circumstances, we need to consider facts happening in other circumstances which are just possible. A material object of this world is soluble now if and only if it would dissolve if it were put in water.

However things could have many other properties and stand in many other relations according to us in possible circumstances (whether actual or not). In addition to the ways in which things are in reality, there are the possible ways in which we can think that they could be. We do not know how the world has been until now and how it will continue. We even ignore most of what is the case now at the present moment. So, as I explained above, we consider a lot of possible truth conditions of atomic propositions different from their actual truth conditions in thinking and speaking. In our mind, the truth of propositions is compatible with many possible ways in which we can represent objects.

However, in order that a proposition be true in a possible circumstance, things must be in that circumstance as that proposition represents them. Otherwise, there would be no correspondence with reality. Along these lines, I propose to define as follows the concept of truth: a proposition is true in a circumstance when its truth in that circumstance is compatible with valuations that assign actual truth conditions to all its atomic propositions. For short, a proposition $P$ is true in a circumstance $m/h$ when a possible valuation $val^+ \in \text{id}_{2P}(m/h)$ is such that $val^+(u_a) = \text{id}_{2P}(u_a)$ for all atomic propositions $u_a \in \text{id}_1P$. As one can expect, the truth of the proposition $P$ in that circumstance is then compatible with all the real valuations of its propositional constituents. In particular any true proposition $P$ is true according to the real valuation $val^*_+$ of atomic propositions. Classical laws of truth theory follow from this concise definition.  

3.1 Cognitive aspects in the theory of truth

A speaker $a$ often rightly or wrongly believes at a moment $m$ that certain propositional constituents could only have such and such possible denotations in circumstances. In that case, atomic propositions composed from such constituents could only be true according to him or her at that moment in such and such sets of possible circumstances. A particular set $Val(a, m)$ of possible valuations of atomic propositions is then compatible with what the speaker $a$
believes at the moment \( m \). Any speaker having in mind atomic propositions believes in the truth of certain propositions containing them. **One can define exactly the notion of truth according to a speaker** in my logic of sense: A proposition is true in a circumstance \( m/h \) according to a speaker \( a \) at a moment \( m \) when the truth of that proposition in that circumstance is compatible with all possible valuations assigned by that agent at that moment to its propositional constituents i.e. when \( \text{Val}(a,m) \subseteq \text{id}_2 \text{P}(m/h) \).

By hypothesis, tautological propositions are true and contradictory propositions are false according to all agents who have them in mind. But impossible propositions which are not contradictory can be true and necessarily true propositions which are not tautological can be false according to agents at some moments. These are basic principles of my epistemic logic. **So the logic of language impose different limits to reality and thought.** On one hand, all propositions which are false in all possible circumstances represent impossible facts that could not exist in reality. (I do not advocate the need in formal semantics of impossible circumstances where such impossible facts could exist.) On the other hand, I admit that there are necessarily false propositions that we can believe (e.g. that whales are fishes). So we can represent impossible facts and wrongly think that they exist. Among necessarily false propositions, I distinguish then those that we can believe from others that we cannot (the pure contradictions). All depends whether their truth is compatible or not with some valuations of their atomic propositions.

**The notion of strong implication**

Human beings are not perfectly rational. We are often inconsistent. Furthermore, we do not make all valid inferences. We assert (and believe) propositions without asserting (and believing) all their logical consequences. Thus, our illocutionary (and psychological) commitments are not as strong as they should be from the logical point of view. However, we, human agents, are not totally irrational. On the contrary, we **manifest a minimal rationality** in thinking and speaking. When we know by virtue of competence that a proposition is false we never relate it to the world with the intention of achieving a success of fit between words and things. So we do not attempt to perform unsatisfiable illocutionary acts with a non empty direction of fit and a contradictory propositional content. Such acts are imperformable because we **a priori** know that they cannot be satisfied.

Moreover, when we **a priori** know by virtue of competence that a proposition cannot be true unless another is also true, we cannot assert (or believe) that proposition without asserting (or believing) the second. There is in philosophical logic an important **relation of strict implication** between propositions that is due to C.I. Lewis. By definition, a proposition strictly implies another proposition whenever that other proposition is true in all possible circumstances where it is true. Hintikka\(^{33}\) and others have advocated that belief and knowledge are

\(^{32}\)The term and notion of minimal rationality comes from Cherniak [1986]

\(^{33}\)See Hintikka Knowledge and Belief Cornell University Press [1962]
closed under strict implication. However, from a cognitive point of view, we ignore how propositions are related by strict implication, just as we ignore in which possible circumstances they are true. Any proposition strictly implies infinitely many necessarily true propositions. However we could not assert (or believe) all of them in any circumstance.

We need a relation of implication much finer than strict implication in order to explicate existing illocutionary and psychological commitments. Thanks to the predicative analysis of the logical form of propositions, one can define a relation of implication that I have called **strong implication** that is finer than all others. A proposition strongly implies another proposition when firstly, it contains all its atomic propositions and secondly, all possible valuations of atomic propositions which are compatible with its truth in a circumstance are also compatible with the truth of that other proposition in that very circumstance. In other words, P strongly implies Q (in symbols P \(\rightarrow\) Q) when \(\text{id}_1 \subseteq \text{id}_2\) and, for any circumstance \(m/h\), \(\text{id}_1(m/h) \subseteq \text{id}_2(m/h)\).

Unlike strict implication, **strong implication is cognitive**. Whenever a proposition P strongly implies another proposition Q we cannot apprehend that proposition P without knowing a priori that it strictly implies the other proposition Q. For in apprehending P, we have by hypothesis in mind all atomic propositions of Q. We make all the corresponding acts of reference and predication. Furthermore, in understanding the truth conditions of proposition P, we consider all possible valuations of these atomic propositions which are compatible with its truth in any circumstance \(m/h\). The same possible valuations of atomic propositions of Q which are in P are then by hypothesis compatible with the truth of proposition Q in that circumstance \(m/h\). Thus, in expressing P, we know that Q follows from P when P strongly implies Q. According to my epistemic logic, belief and knowledge are then closed under strong rather than strict implication.

As I will show, strong implication obeys a series of important universal laws. Unlike strict implication, strong implication is anti-symmetrical. Two propositions which strongly imply each other are identical. Unlike Parry’s analytic implication, strong implication is always tautological. In my terminology, a proposition P tautologically implies another Q when, for any circumstance \(m/h\), \(\text{id}_1(m/h) \subseteq \text{id}_2(m/h)\). Natural deduction rules of elimination and introduction generate strong implication when and only when all atomic propositions of the conclusion belong to the premises. So a proposition P does not strongly imply any disjunction of the form P \(\lor\) Q. Moreover strong implication is para-consistent. A contradiction does not strongly imply all propositions. Finally, strong implication is finite and decidable. (More on this later)

4 ANALYSIS OF MODAL AND TEMPORAL PROPOSITIONS

Let us now analyze in terms of predication the logical form of modal and temporal propositions in the framework of the logic of ramified time. We need to
take into account the following facts:

1. **As regards their structure of constituents**

   Unlike truth functions, modal and temporal operations on propositions enrich the set of their atomic propositions. As I said earlier, we predicate modal and temporal attributes in expressing modal and temporal propositions. For example, in asserting that Paul was previously married to Julie we predicate of Paul the temporal property of being an ex-husband of Julie.

2. **As regards their truth conditions**

   As Occam already pointed out, in order to analyze the truth conditions of future propositions we need to consider not only moments of time but also histories. In the logic of branching time, a *moment* is a possible complete state of the world at a certain instant and the *temporal relation of anteriority / posteriority* between moments is partial rather than linear because of indeterminism. On the one hand, there is a single causal route to the past: each moment $m$ is preceded by at most one past moment $m'$. And all moments are historically connected: any two distinct moments have a common historical ancestor in their past. On the other hand, there are multiple future routes: several incompatible moments might follow upon a given moment. For facts existing at a moment can have incompatible future effects.

   Consequently, the set of moments of time in any interpretation has the formal structure of a *tree-like frame* which can be represented as follows:

![Tree-like frame diagram]

A maximal chain $h$ of moments of time is called a *history*. It represents a possible course of history of the actual world. There are nine different histories in the preceding figure. A first history $h_1$ goes from moment $m_0$ to moment $m_7$, a second one $h_2$ from moment $m_0$ to moment $m_8$ and so on. The truth of certain propositions is settled at each moment no matter which historical continuation of that moment is under consideration. So are past propositions because all histories through a moment have the same past at that moment. The past proposition that it was the case that $A$ (in symbols: $WasA$) is true at a moment $m$ according to any history when $A$ is true at a moment $m'$ anterior...
to \( m \). Thanks to histories, branching temporal logic can analyze the notion of \textit{settled truth}. The proposition that it is settled that \( A \) (in symbols \( \text{Settled}A \)) is true at a moment \( m \) according to a history \( h \) if and only if the proposition that \( A \) is true at that moment \( m \) according to all histories to which it belongs. Unlike what is the case for past propositions, the truth of future propositions is not settled at each moment; it depends on which historical continuation \( h \) of that moment is under consideration. Following Peirce and Belnap [1994] I will say that the future proposition that it will be the case that \( A \) (in symbols \( \text{Will}A \)) is true at a moment \( m \) according to a history \( h \) when the proposition that \( A \) is true at a moment \( m' \) posterior to \( m \) according to that very history \( h \).

Given the causal ordering relation, some histories \( h \) and \( h' \) are \textit{undivided} at certain moments \( m \); they have the same present and past at these moments. In that case, moment \( m \) and all moments \( m' \) anterior to that moment belong to both histories \( h \) and \( h' \). The relation of having the same present and past at a moment \( m \) is an equivalence relation which partitions the set of histories to which \( m \) belongs into a family of exhaustive and pairwise disjoint subsets, each of which keeps undivided histories at the next moment together. Each set of histories in the partition is an \textit{elementary immediate possibility} after \( m \). If there is only one such subset in the partition, the moment \( m \) is \textit{deterministic}. Otherwise, it is \textit{undeterministic}.

Two moments of time are \textit{alternative} in my terminology when they belong to histories which have the same past before these moments. For example, moments \( m_7, m_8 \) and \( m_9 \) are alternative in the last figure. They represent how the world could be immediately after the moment \( m_3 \). The set \textit{Instant} of all instants is a partition of the set \textit{Time} of all moments of time which contains exactly one moment of each history and respects the temporal order of histories. Moments which belong to the same instant are said to be \textit{coinstantaneous}. So the first instant is the singleton containing the moment of time which is anterior to all others (such a first moment exists in an interpretation if the world has a beginning according to it). And after each instant \( \iota \) the next instant \( \iota' \) is the set that contains all and only the alternative moments that follow the moments of that instant \( \iota \). For example, moments \( m_3, m_4, m_5 \) and \( m_6 \) of figure 1 are coinstantaneous.

Thanks to instants, branching logic can analyze important modal notions such as \textit{historic necessity} (in the sense of now unpreventability)\textsuperscript{36} and \textit{historic possibility}. Consider the proposition that it is then necessary that \( A \) (in sym-

\textsuperscript{34}According to the actualist point of view (that Occam was the first to advocate), the future proposition \( \text{Will}A \) is rather true at a moment \( m \) if and only if the proposition that \( A \) is true at a moment \( m' \) posterior to \( m \) according to the particular history that represents the actual historic continuation of that moment. See Belnap [2001]’s arguments against actualism.

\textsuperscript{35}We do not know how the world will continue. But among all alternative moments that could directly follow the present moment we know that one and only one will be actual (if the world continues). So among all alternative moments that follow immediately any moment \( m \) a single one would belong to the actual history of this world if that moment \( m \) were actual.

\textsuperscript{36}As Prior [1967] says, now unpreventable propositions are “those outside our power to make true or false”
bols □A) in the sense that it could not have been otherwise than A: it is true at a moment \( m \) according a history \( h \) when the proposition that A is true at all moments \( m' \) coinstantaneous with \( m \) according to the histories \( h' \) to which they belong. Whenever □A is true at a moment \( m \), A represents a fact that is not only settled but also \textit{inevitable} at that moment. Similarly, the proposition that it is then possible that A (in symbols ♦A) is true at a moment \( m \) according a history \( h \) when the proposition that A is true at some moment \( m' \) coinstantaneous with \( m \) according to at least one history \( h' \).

\textbf{The ideal object language}

The ideal object propositional language \( L \) of my logic of ramified time and historic modalities contains the following syntactic resources:

\textbf{Vocabulary of \( L \)}

Language \( L \) contains in its lexicon:

1. a series of propositional symbols \( p, p', p'', \ldots, q, q', q'', \ldots \)
2. the syncategorematic expressions:
   - Tautological
   - \( > \), \( \land \), \( \square \), \textit{Will}, \textit{Was}, \( \neg \), ( and ).

\textbf{Rules of formation of \( L \)}

The set \( L_p \) of propositional formulas

Propositional symbols are propositional formulas. If \( A_p \) and \( B_p \) are propositional formulas then \( \neg A_p \), \( \square A_p \), \textit{Will} \( A_p \), \textit{Was} \( A_p \), Tautological(\( A_p \)), \( (A_p > B_p) \) and \( (A_p \land B_p) \) are new complex propositional formulas.

Propositional symbols express propositions.

A formula of the form \( \neg A_p \) expresses the negation of the proposition expressed by \( A_p \).

\( \square A_p \) expresses the modal proposition that \( A_p \) is then necessary (i.e. that it could not have been otherwise than \( A_p \)).

\textit{Will} \( A_p \) expresses the future proposition that it will be the case that \( A_p \).

\textit{Was} \( A_p \) expresses the past proposition that it has been the case that \( A_p \).

Tautological(\( A_p \)) expresses the proposition that \( A_p \) is tautological.

\( (A_p > B_p) \) expresses the proposition that all atomic propositions of \( B_p \) are atomic propositions of \( A_p \).

\( (A_p \land B_p) \) expresses the conjunction of the two propositions expressed by \( A_p \) and \( B_p \).

\textbf{Rules of abbreviation}

Exterior parentheses will often be omitted. Parentheses will also be omitted according to the rule of the association to the left. Truth, modal and temporal connectives are introduced according to usual rules.

\textit{Disjunction:} \((A_p \lor B_p) =_d f \neg(\neg A_p \land \neg B_p) \) whenever \( A \) and \( B \) \( \in L_p \)

\textit{Material implication:} \((A_p \Rightarrow B_p) =_d f \neg(A_p \land \neg B_p) \)

\textit{Material equivalence:} \((A_p \Leftrightarrow B_p) =_d f (A_p \Rightarrow B_p) \land (B_p \Rightarrow A_p) \)

\textit{Was-always} \( A_p =_d f \neg\textit{Was} \neg A_p \)

\textit{Will-always} \( A_p =_d f \neg\textit{Will} \neg A_p \)

\textit{Always} \( A_p =_d f \textit{Was-always} A_p \land A_p \land \textit{Will-always} A_p \)

\textit{Sometimes} \( A_p = \textit{Was} A_p \lor A_p \lor \textit{Will} A_p \)

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Historical possibility: ♦Ap =df ¬□¬Ap

Here are new abbreviations:
• Analytic implication
• Strong implication
• Same structure of constituents
• Propositional identity

5 THE FORMAL SEMANTICS

One must interpret formulas of L and associate with them truth conditions according to the following rules. A standard model for L is a quintuple M of the form

<Time , Instant , Atom , Val , |||>, where

(1) Time is a non empty set whose elements m, m’, m”... are moments which represent possible complete states of the world. ≤ is a partial order on the set Time representing the causal ordering relation or the temporal relation of anteriority / posteriority. m < m’ means that moment m is in the past of moment m’ and that moment m’ is in the future of possibilities of m. By definition, < is subject to historical connection and no downward branching. Any two distinct moments m and m’ have a common historical ancestor: some moment m” such that m” < m and m” < m’. Moreover, the past is unique: if there is a moment m” such that m < m” and m’ < m” then either m = m’ or m < m’ or m’ < m.

Consequently, (Time , =) is a tree-like frame. A maximal chain h of moments of Time is called a history. It represents a possible course of history of the world. Let History be the set of all histories. Two histories h and h’ are undivided at a moment m when they have the same present and past at these moments, that is to say when, for all moments m’ ≤ m, m’ ∈ h and m’ ∈ h’.

(2) The set Instants, whose elements i, i’,... are called instants, is a partition of the set Time which satisfies unique intersection and order preservation. So for all i and h there is a unique moment m (in symbols m(i,h)) belonging to i and h. And m(i, h) ≤ m(i’, h’) when m(i, h’) ≤ m(i’, h’). Two moments of time m and m’ are cointantaneous (in symbols: m ≡ m’) when they belong to the same instant. Any pair of cointantaneous moments m and m’ represent two complete possible states of the world in which things could be at a certain instant.

(3) Atoms is an infinite set whose elements are atomic propositions. In the present logic, atomic propositions are left undefined. However, as I have explained earlier, one can define their formal nature using the modal theory of types.

37 The notion of analytic implication comes from W.T. Parry [1933] and [1972]. See also K. Fine [1986].
\( \mathbf{P} \) \([\text{Atoms}]\) is an upper modal temporal and agentive semi lattice containing finite sets of atomic propositions which is closed under union \( \cup \) and a unary modal and temporal operation \( ^* \) satisfying the following conditions: For any \( \Gamma_a \subseteq \text{Atoms} \), \( \Gamma_a \subseteq ^*(\Gamma_a) \) and for any \( \Gamma_1 \) and \( \Gamma_2 \subseteq \text{Atoms} \), \( ^*(\Gamma_1 \cup \Gamma_2) = ^*(\Gamma_1) \cup ^*(\Gamma_2) \) and \( ^*^*(\Gamma_1) = ^*(\Gamma_1) \).\(^{38}\)

All the elements of \( \mathbf{P} \) \([\text{Atoms}]\) are finite sets of atomic propositions from which expressible propositions can be composed.

(4) \( \text{Val} \) is the set of all functions from \( \text{Atoms} \) into \( \mathbf{P} \) \([\text{Time} \times \text{History}]\). Its elements are \emph{valuations of atomic propositions}. Each valuation \( \text{val} \in \text{Val} \) assigns possible truth conditions to atomic propositions. \( m, h \in \text{val}(u_a) \) means that atomic proposition \( u_a \) is true at moment \( m \) according to history \( h \) under that valuation. When \( m, h \in \text{val}(u(a)) \), \( m \in h \). I will often indicate this by writing \( m/h \). There is a distinguished valuation \( \text{val}_M \in \text{Val} \) determining the actual truth conditions that atomic propositions have under the model \( M \). So, for any atomic proposition \( u_a \), \( \text{val}_M(u_a) \) is the set of all pairs \( m/h \) of moments and histories where that atomic proposition is true according to the model \( M \).

(5) The set \( \text{U}_p \) of all \emph{propositions} which are expressible in \( L \) according to \( M \) is an infinite subset of the Cartesian product \( \mathbf{P} \) \([\text{Atoms}]\) \( \times \) \( \text{Val} \) \([\text{Time} \times \text{History}]\). The first term, \([P] \), of a proposition \( P \) represents the set of its atomic propositions. And its second term, \( |P| \), its truth conditions. So for each moment \( m \) and history \( h \), where \( m \in h \), \( |P|_{m/h} \) is the set containing all possible valuations of atomic propositions which are compatible with the truth of proposition \( P \) at the moment \( m \) according to the history \( h \).

(6) \( |\ | \) is an interpreting function which associates with each propositional formula the proposition that that formula expresses according to model \( M \). The proposition \( |\text{A}_p| \) expressed by formula \( \text{A}_p \in L_p \) in the model \( M \) is a pair \(< \text{[A}_p|] , |\text{A}_p| > \) belonging to the Cartesian product \( \mathbf{P} \) \([\text{Atoms}]\) \( \times \) \( \text{Val} \) \([\text{Time} \times \text{History}]\).

Proposition \( |\text{A}_p| \) is defined inductively as follows:

(i) For any propositional symbol \( p \), \( |p| \in \text{U}_p \).
(ii) \([\text{TautologicalB}_p] = [B_p] \) and \([\text{TautologicalB}_p|_{m/h} = \text{Val} \) when \( [B_p|_{m/h} = \text{Val} \). Otherwise, \([\text{TautologicalB}_p|_{m/h} = \emptyset \).
(iii) \([-B_p] = [B_p] \) and \([-B_p|_{m/h} = \text{Val} - [B_p|_{m/h} \).
(iv) \([\Box B_p] = [B_p|_{m/h} \) and \([\Box B_p|_{m/h} = \bigcup_{m'} \{[B_p|_{m'/h} \text{ where } m' \equiv m \} \).
(v) \([\text{WillB}_p] = [B_p] \) and \([\text{WillA}_p|_{m/h} = \bigcup_{m'>m} [A_p|_{m'/h} \).
(vi) \([\text{WasB}_p] = [B_p] \) and \([\text{WasA}_p|_{m/h} = \bigcup_{m'<m} [A_p|_{m'/h} \).

\(^{38}\)Suppose the set \( \Gamma \subseteq U_a \) contains an atomic proposition \( u_a \) predicating an attribute \( R \) in a certain order of \( n \) objects under concepts. In a propositional logic with attributes, the new set \( ^*(\Gamma) \) contains three new atomic propositions predicating respectively the historic necessitation \( \Box R \), the temporal posteriorization \( \text{WillR} \) and anteriorization \( \text{WasR} \) of that attribute of the same objects in the same order. These modal and temporal attributes have the following extensions:

For any \( u_1 \ldots u_n \in U_c \), \( < u_1 \ldots u_n > \in \Box R \) \((m/h) \) iff \( \langle u_1 \ldots u_n \rangle \in R \) \((m'/h') \) for all moments \( m' \) coinstantaneous with \( m \) and histories \( h' \). Moreover, \( < u_1 \ldots u_n > \in \text{WillR} \) \((m/h) \) iff, for at least one \( m' > m \), \( < u_1 \ldots u_n > \in R \) \((m'/h) \) and similarly for \( \text{WasR} \) \((m/h) \) except that \( m' < m \).
(vii) \([B_p \land C_p] = [B_p] \cup [C_p]\) and \([B_p \land C_p|_{m/h}] = [B_p|_{m/h} \land [C_p|_{m/h}].\)
(viii) \([B_p \geq C_p] = [B_p] \cup [C_p]\) and \([B_p \geq C_p|_{m/h}] = \text{Val}\) when \([B_p] \geq [C_p].\)
Otherwise \([B_p \geq C_p|_{m/h}] = \emptyset.\)

**Definition of truth and validity**

A proposition \(\|A_p\|\) is true at a moment \(m\) according to a history \(h\) under the model \(M\) when the real valuation \(\text{val}_M \in [A_p]_{m/h}.\) A formula \(A_p\) of \(L\) is valid or logically true (in symbols: \(\models A_p\)) when \(\|A_p\|\) is true at all moments according to all histories in all standard models \(M\) of \(L.\)

**6 A COMPLETE AXIOMATIC SYSTEM**

I conjecture that all and only the valid formulas of my logic are provable in the following axiomatic system \(S.\)

The axioms of \(S\) are all the instances in \(L\) of the following axiom schemas:

**Classical truth functional logic**

- (T1) \((A_p \Rightarrow (B_p \Rightarrow A_p))\)
- (T2) \(((A_p \Rightarrow (B_p \Rightarrow C_p)) \Rightarrow ((A_p \Rightarrow B_p) \Rightarrow (A_p \Rightarrow C_p)))\)
- (T3) \(((\neg A_p \Rightarrow \neg B_p) \Rightarrow (B_p \Rightarrow A_p))\)

**S5 Modal logic**

- (T4) \((\Box A_p \Rightarrow A_p)\)
- (T5) \((\Box (A_p \Rightarrow B_p) \Rightarrow \Box (A_p \Rightarrow \Box B_p))\)
- (T6) \((\Diamond A_p \Rightarrow \Box \Diamond A_p)\)

**Axioms for tautologies**

- (T7) \((\text{Tautological}(A_p)) \Rightarrow \Box A_p\)
- (T7) \((\text{Tautological}(A_p)) \Rightarrow (A_p = (A_p \Rightarrow A_p))\)
- (T7) \((\neg \text{Tautological}(A_p)) \Rightarrow ((\text{Tautological}(A_p)) = (A_p \land \neg A_p))\)
- (T8) \(\text{Tautological}(A_p) \Rightarrow (\text{Tautological}(A_p \Rightarrow B_p) \Rightarrow \text{Tautological}(B_p))\)
- (T9) \(\text{Tautological}(A_p) \Rightarrow \text{Tautological}(\Box \Box A_p)\)
- (T10-11) \(\text{Tautological}(A_p) \Rightarrow \text{Tautological}(\text{Will-always} A_p)\) And similarly for \(\text{Was-always}\)
- (T12) \((A_p = B_p) \Rightarrow \text{Tautological}(C \Rightarrow C^*)\) where \(C^*\) and \(C\) are propositional formulas which differ at most by the fact that an occurrence of \(B_p\) replaces an occurrence of \(A_p\) whenever there is no confusion of bound variables.

**Axioms for propositional composition**

- (C1) \((A_p \times B_p) \Rightarrow \text{Tautological}(A_p \times B_p)\)

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39For the sake of simplicity, I have interpreted the logical constants \(\text{Tautological}\) and \(\geq\) as simple propositional connectives that only rearrange truth conditions rather than second-order predicates expressing attributes of propositions. According to this interpretation they do not change the structure of constituents. Consequently, all propositional formulas express first order propositions in my formal semantics.
(C2) \( \neg(A_p > B_p) \Rightarrow Tautological\neg(A_p > B_p) \)

(C3) \( A_p > A_p \)

(C4) \( (A_p > B_p) \Rightarrow ((B_p > C_p) \Rightarrow (A_p > C_p)) \)

(C5) \( (A_p \land B_p) > A_p \)

(C6) \( (A_p \land B_p) > B_p \)

(C7) \( (C_p > A_p) \Rightarrow ((C_p > B_p) \Rightarrow (C_p > (A_p \land B_p))) \)

(C8) \( A_p \equiv \neg A_p \)

(C9) \( \square A_p \rightarrow A_p \)

(C10) \( \square \neg A_p \equiv \neg A_p \)

(C11) \( \square (A_p \land B_p) \equiv (\square A_p \land \square B_p) \)

(C12) \( \square \square A_p \equiv \square A_p \)

(C13) \( Will A_p \equiv \square A_p \)

(C14) \( Was A_p \equiv \square \square A_p \)

Branching time logic

(TL1) \( (Will-always(A_p \Rightarrow B_p) \Rightarrow (Will-always A_p \Rightarrow Will-always B_p)) \)

(TL2) \( (Was-always(A_p \Rightarrow B_p) \Rightarrow (Was-always A_p \Rightarrow Was-always B_p)) \)

(TL3) \( (A_p \Rightarrow Was-always Will A_p) \)

(TL4) \( (A_p \Rightarrow Will-always Was A_p) \)

(TL7) \( (Was A_p \Rightarrow Will-always Was A_p) \)

(TL8) \( (Will A_p \Rightarrow Will-always Will A_p \land A_p \lor Was A_p) \)

(TL9) \( (Was A_p \Rightarrow Was-always (Will A_p \lor A_p \lor Was A_p) \)

Historic modality with time

(MT1) \( (Was \square (A_p \land Will-always B_p) \land Was-always \neg (B_p \land \Diamond C_p)) \Rightarrow \square (Will-always D_p \land Was C_p \Rightarrow Was (A_p \land (C_p \lor Was C_p) \land Will-always (C_p \Rightarrow Will-always D_p))) \)

(MT2) \( (Was-always (A_p \land Was-always \neg (B_p \land \Diamond C_p) \land Will (B_p \land A_p \land \Diamond D_p)) \land Was (\square E_p \land Will-always B_p)) \Rightarrow \square (Will-always Q_p \Rightarrow Was (E_p \land Will-always (C_p \Rightarrow Will-always D_p \Rightarrow Will-always Q_p))))^{40} \)

The rules of inference of axiomatic system S are:

The rule of Modus Ponens: From the sentences \( A \Rightarrow B \) and \( A \) infer \( B \).

The necessitation rules: From a theorem \( A \) infer \( Tautological A \).

7 VALID LAWS

7.1 Laws of composition

A proposition is composed from all the atomic propositions of its constituent propositions.

Thus \( \models A_p > p \) when \( p \) occurs in \( A_p \).

There is a law of distribution of the constituent atomic propositions of modal and temporal propositions with respect to truth functions. \( \models M(A_p \land B_p) \equiv (MA_p \land MB_p) \)

^{40} Axions (MT1-2) are A. Zanardo [1985]'s axioms of local correspondence.
where M is of the form □, ¬□, □¬, ¬□¬, Will¬, Was, Was¬, ¬Will or ¬Was. So all the different modal and temporal propositions of the form MA_p have the same atomic propositions.

\[ \vdash MA_p \equiv M'A_p \], where M and M’ are □, ¬□, □¬, ¬□¬, Will¬, Was, Was¬, ¬Will or ¬Was.

### 7.2 Laws for tautologyhood

There are modal, temporal as well as truth functional tautologies. Thus \( \vdash \text{Tautological}(\Box A_p \Rightarrow A_p) \) and \( \vdash \text{Tautological}(A_p \Rightarrow \text{Will-always Was} A_p) \).

Tautologyhood is stronger than logical necessity. Thus \( \not\vdash \text{Tautological}(A_p) \Rightarrow \Box A_p \).

But \( \not\vdash \Box A_p \Rightarrow \text{Tautological}(A_p) \). Necessarily true elementary propositions like the proposition that whales are mammals are not tautological.

### 7.3 Laws for tautological implication

Tautological implication is finer than strict implication.

\( \vdash \text{Tautological}(A_p \Rightarrow B_p) \Rightarrow (A_p \in B_p) \) But the converse is not true. For example, the elementary proposition that the biggest whale is a fish strictly implies the contradiction that it is and that it is not a fish. But it does not tautologically imply that contradiction. Only elementary propositions whose atomic proposition is it self contradictory tautologically imply a contradiction. Whenever a proposition tautologically implies another, we can express the first without expressing the second. (Suppose the second contains new atomic propositions.) But one cannot have both in mind without knowing that the first imply the second.

### 7.4 Laws for strong implication

Strong implication is the strongest kind of propositional implication. By definition, \( A_p \rightarrow B_p = (A_p > B_p) \land \text{Tautological} (A_p \Rightarrow B_p) \)

Whenever a proposition strongly implies another proposition, this is tautological. \( \vdash (A_p \rightarrow B_p) \leftrightarrow \text{Tautological} (A_p \rightarrow B_p) \).

Any proposition implying strongly another proposition is identical with its conjunction with that other proposition. So \( \vdash (A_p \rightarrow B_p) \leftrightarrow ((A_p \land B_p) = A_p) \)

There are two causes of failure of strong implication:

Firstly, \( \vdash \neg (A_p > B_p) \Rightarrow \neg (A_p \rightarrow B_p) \). A proposition does not strongly imply any proposition composed from other atomic propositions: In that case, it is possible to have in mind the first proposition without having in mind the second.

Secondly, \( \vdash \neg \text{Tautological}(A_p \Rightarrow B_p) \Rightarrow \neg (A_p \rightarrow B_p) \). A proposition does not strongly imply any proposition that it does not tautologically imply. In that case, we do not necessarily know by virtue of linguistic competence that the first proposition has more truth conditions than the second.
Unlike strict implication, strong implication is a relation of partial order. So Parry’s analytic implication, which is not anti-symmetric, is weaker than strong implication. \( \models (A_p \leftrightarrow B_p) \Rightarrow (A_p \rightarrow B_p) \). But \( \not\models (A \rightarrow B) \Rightarrow (A \leftrightarrow B) \). For \( \not\models (A \rightarrow B) \Rightarrow Tautological(A \Rightarrow B) \).

Paradoxical laws of the following kind do not hold for strong implication:
\[ \models ((□p \vee
\neg □p) \rightarrow □p) \vee (□p \rightarrow (□p \wedge \neg □p)) \]

\section*{7.5 Natural deduction}

All and only the valid laws of inference of modal and temporal logic where the premises contain all atomic propositions of the conclusion are valid laws of strong implication. This leads to the following system of natural deduction for strong implication:

The law of elimination of conjunction:
\[ \models (A_p \wedge B_p) \rightarrow A_p \quad \text{and} \quad \models (A_p \wedge B_p) \rightarrow B_p \]

The law of elimination of disjunction:
\[ \models ((A_p \rightarrow C_p) \wedge (B_p \rightarrow C_p)) \Rightarrow (A_p \vee B_p) \rightarrow C_p \]

Failure of the law of introduction of disjunction: \( \not\models A_p \rightarrow (A_p \vee B_p) \)
So strong implication is stronger than entailment in the sense of the logic of relevance. For the law of introduction of disjunction holds for entailment.

Failure of the law of elimination of negation:
\[ \not\models (A_p \wedge \neg A_p) \rightarrow B_p \]

Strong implication is paraconsistent.

The law of elimination of material implication:
\[ \models (A_p \wedge (A_p \Rightarrow B_p)) \rightarrow B_p \]

The law of elimination of necessity:
\[ \models □A_p \rightarrow A_p \]

The law of elimination of always:
\[ \models \text{Always}A_p \rightarrow A_p \]

The law of introduction of necessity:
\[ \models (A_p \rightarrow B_p) \Rightarrow (□A_p \rightarrow □B_p) \]

And similarly for Will, Was.

Failure of the law of elimination of possibility:
\[ \not\models (\diamond A_p \rightarrow B_p) \Rightarrow A_p \rightarrow B_p \]

Failure of the law of introduction of possibility:
\[ \not\models A_p \rightarrow \diamond A_p \]

For \( A_p \rightarrow \diamond A_p \). And similarly for Sometimes.

Strong implication is decidable. For \( \models A_p \rightarrow B_p \) when all propositional symbols which occur in B also occur in A. Moreover, \( \models Tautological(A_p \Rightarrow B_p) \) when semantic tableau for \( (A_p \Rightarrow B_p) \) close.\[41\]

There is also a theorem of finiteness for strong implication:

Every proposition only strongly implies a finite number of other propositions. In particular, a proposition strongly implies all and only the tautologies which are composed from its atomic propositions.
\[ \models Tautological B_p \Rightarrow (A_p \rightarrow B_p \Leftrightarrow A_p \rightarrow B_p) \]

Similarly, a contradiction strongly implies all and only the propositions composed from its atomic propositions. \( \models Tautological \neg A_p \Rightarrow (A_p \rightarrow B_p \Leftrightarrow (A_p \rightarrow B_p)) \)

The fact that knowledge is closed under strong implication is confirmed by the decidability and finiteness of strong implication.

\[41\] The present philosophical logic is decidable.
7.6 Laws of propositional identity

Modal and temporal propositions are composed from several atomic propositions. So \( \not\equiv (\square A_p = p) \) for any propositional symbol \( p \). And similarly for \( \text{Will} \) and \( \text{Was} \).

The failure of this law is shown in language use. Properties such as being identical with itself are possessed by all objects in all circumstances. They have the same extension as their necessitation. But when we think that Oedipus is identical with himself, we do not eo ipso think that it is necessary that he be identical with himself. So modal propositions are not reducible to elementary propositions.

All the classical Boolean laws of idempotence, commutativity and associativity remain valid:

\[ \models A_p = (A_p \land A_p) \]
\[ \models (A_p \land B_p) = (B_p \land A_p) \]
\[ \models (A_p \land (B_p \land C_p)) = ((A_p \land B_p) \land C_p) \]

As well as the laws of distributivity:

\[ \models \neg(A_p \lor B_p) = (\neg A_p \land \neg B_p) \]
\[ \models A_p \land (B_p \lor C_p) = (A_p \land B_p) \lor (A_p \land C_p) \]
\[ \models \square(A_p \land B_p) = (\square A_p \land \square B_p) \]

And the laws of reduction:

\[ \models \neg \neg A_p = A_p \]
\[ \models M \square A_p = \square A_p \]
\[ \models M \Diamond A_p = \Diamond A_p \]

Where \( M = \square, \boxminus, \diamondsuit \) or \( \Box \downarrow \).

Identical propositions need not be intensionally isomorphic in the sense of hyperintensional logic. As I have argued in [1990-91], intensional isomorphism is a too strong criterion of propositional identity.

However, propositional identity requires more than co-entailment in the sense of the logic of relevance. For \( \not\equiv A \equiv (A \land (A \lor B)) \), M. Dunn regrets that \( A \) and \( (A \land (A \lor B)) \) co-entail each other. For most formulas of such forms are not synonymous. Co-entailment is not sufficient for synonymy because it allows for the introduction of new senses. Strong equivalence which requires the same structure of constituents is necessary for an adequate analysis of synonymy. Finally, strong equivalence is finer than Parry’s analytic equivalence. \( \not\equiv (\square p) \Rightarrow (\square p = (\square p \lor \neg \square p)) \) and \( \not\equiv (\neg \square p) \Rightarrow (\neg \square p = (\square p \land \neg \square p)) \). But such paradoxical laws hold for analytic equivalence.

Notice that the law of determinism does not hold in the logic of branching time.

So \( \not\equiv A_p \Rightarrow \text{Was-always} \square \text{Will} A_p \) and \( \not\equiv \square A_p \Rightarrow \text{Was-always} \square \text{Will} A_p \). Similarly, \( \not\equiv \text{Will} A_p \Rightarrow \square \text{Will} A_p \). But the following new laws hold for historic modalities:

\[ \models (\text{Was} \square A_p \Rightarrow \square \text{Was} \square A_p) \]
\[ \models (\square \text{Will-always} A_p \Rightarrow \text{Will-always} \square A_p) \]

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42 See Max J. Cresswell [1975]
43 See his Philosophical Rumifications in Anderson et al [1992]
44 I thank Nuel Belnap for having drawn my attention to these laws.
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