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**SRAFFA AND WITTGENSTEIN :  
PHYSICALISM AND CONSTRUCTIVISM**

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*Is it already mathematical alchemy, that  
mathematical propositions are regarded as  
statements about mathematical objects, – and  
mathematics as the exploration of these objects?*

*L. Wittgenstein*

1. Piero Sraffa is the subject of much curiosity because of his friendship with two other major figures of the 20<sup>th</sup> century – an unlikely pair –, Antonio Gramsci and Ludwig Wittgenstein. Amartya Sen recently speculated about a “Gramsci connection”, with Sraffa as the intermediary, in the evolution of Wittgenstein’s thought (Sen 2003, 1241; 2004, 25).<sup>1</sup> With the exception of a few remarks on Gramsci at the beginning of the next section, I shall be concerned with the connection between Sraffa and Wittgenstein. As no other than Paul Samuelson wrote: “one yearns to know more about Sraffa’s precise influence on Wittgenstein” (Samuelson 1990, p. 264).

From 1927 onward, Sraffa lived in Cambridge, to which Wittgenstein came back in 1929. Sraffa joined him at Trinity in 1939, so they were colleagues until Wittgenstein’s death in 1951.<sup>2</sup> They met early on, in 1929, through the intermediary of John Maynard Keynes. Cambridge must have been an unusually stimulating milieu. One anecdote has Ramsey, Sraffa, Wittgenstein, and Keynes discussing the latter’s *Treatise on Probability* over lunch (Newman 1987, 42). Sraffa and Wittgenstein had opportunity to meet throughout most of the 1930s and 1940s. It appears that they met frequently, at times more than once a week, until May 1946, when he told Wittgenstein that he no longer wished to have conversations with him. Wittgenstein’s biographer, Ray Monk, reported the incident in these terms:

In May 1946 Piero Sraffa decided he no longer wished to have conversations with Wittgenstein, saying that he could no longer give his time and attention to the matters Wittgenstein wished to

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<sup>1</sup> On this, see also (Davis 1993) and (Davis 2002).

<sup>2</sup> For biographical information about Sraffa, see (Kaldor 1985) and (Potier 1991); for Wittgenstein, see (Monk 1991).

discuss. This came as a great blow to Wittgenstein. He pleaded with Sraffa to continue their weekly conversations, even if it meant staying away from philosophical subjects. ‘I’ll talk about anything’, he told him. ‘Yes’, Sraffa replied, ‘but in your way’. (Monk 1991, 487)

Sraffa must have been an exceptional conversationalist; Georg Henrik von Wright reported Wittgenstein as saying that “discussions with Sraffa made him feel like a tree from which all branches had been cut. That this tree could become green again was due to its own vitality” (Malcolm 1984, 14-15). Discussions with Sraffa had a great impact on Wittgenstein. Regarding mistakes he thought he had made in his *Tractatus Logico-Philosophicus* (hereafter shortened to *Tractatus*) he wrote:

I was helped to realize these mistakes, to a degree which I myself am hardly able to estimate, by the criticism which my ideas encountered from Frank Ramsey, with whom I discussed them in the last two years of his life. Even more than to this always certain and forcible criticism I am indebted to that which a teacher at this University, Mr. P. Sraffa, for many years unceasingly practiced on my thoughts. I am indebted to this stimulus for the most consequential ideas of this book. (Wittgenstein 1953, viii)

There is also an earlier, well-known passage, dating from 1932, where Wittgenstein recognized his debt to Sraffa, as the last of an impressive list:

I don’t believe I have ever invented a line of thinking, I have always taken one over from someone else. I have simply straightaway seized on it with enthusiasm for my work of clarification. That is how Boltzmann, Hertz, Schopenhauer, Frege, Russell, Kraus, Loos, Weininger, Spengler, Sraffa have influenced me. (Wittgenstein 1980, 19)

In these passages, Wittgenstein states that he owes “the most consequential ideas of this book” from discussions with Sraffa and that he took a “line of thinking” from him. Alas, the precise nature of this “stimulus” remains subject to speculation. The reason for this is that there are hardly any traces of their conversations.<sup>3</sup> There is only one recorded instance, an oft-quoted anecdote reported by Norman Malcolm:

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<sup>3</sup> The Sraffa papers are deposited at the Wren Library, Trinity College, Cambridge. The catalogue is available at <http://rabbit.trin.cam.ac.uk/~jon/Sraffa/Sraffaframeset.html>. For information about their content, see (Kurz 1998). Among these papers, one only finds ‘Notes on Wittgenstein’s *Blue Book*, correspondence with G H von Wright and news cuttings of reviews of books by and on Wittgenstein’ (catalogue No. I21). There is also a ‘Fragment on Language’ [D3/12/7], the content of which bears affinity with ideas from Wittgenstein. As for Wittgenstein, almost all references to Sraffa in his *Nachlaß* are cited here. The only published letter, from Sraffa to Wittgenstein,

Wittgenstein and P. Sraffa [...] argued together a great deal over the ideas of the *Tractatus*. One day (they were riding, I think, on a train) when Wittgenstein was insisting that a proposition and that which it describes must have the same ‘logical form’, the same ‘logical multiplicity’, Sraffa made a gesture, familiar to Neapolitans as meaning something like disgust or contempt, of brushing the underneath of his chin with an outward sweep of the finger-tips of one hand. And he asked: ‘What is the logical form of *that*?’ Sraffa’s example produced in Wittgenstein the feeling that there was an absurdity in the insistence that a proposition and what it describes must have the same form. This broke the hold on him of the conception that a proposition must literally be a ‘picture’ of the reality it describes.” (Malcolm 1984, 57-58)

There are also a handful of references to conversations with Sraffa in Wittgenstein’s manuscripts, such as this one:<sup>4</sup>

Are the propositions of mathematics anthropological propositions saying how we men infer and calculate? – Is a statute book a work of anthropology telling how the people of this nation deal with a thief etc.? – Could it be said: “The judge looks up a book about anthropology and thereupon sentences the thief to a term of imprisonment”? Well, the judge does not USE the statute book as a manual of anthropology. (Discussion with Sraffa) (Wittgenstein 1978, III, par. 65)<sup>5</sup>

On the basis of evidence such as this, commentators had to use ingenuity to come up with plausible suggestions. Wittgenstein commented to Rhus Rhees that the most important thing he gained from his discussions with Sraffa was an ‘anthropological’ way of looking at things (Monk 1991, 261). This is not self-explanatory but, if by ‘anthropological’ one understands, as Ray Monk did, looking at language not in isolation from circumstances in which it is used, as Wittgenstein did in his *Tractatus*, but within the context of the activities within which it is embedded, then, surely, Sraffa’s influence on the later Wittgenstein is “indeed of the most

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14/3/1936 (Wittgenstein 1995, 290-292), interesting as it may be, has also no philosophical content. Further letters from Wittgenstein to Sraffa were recently acquired by Trinity College. I have not been able to see the letters but was informed about their content by Jonathan Smith, archivist at Trinity, and Carlo Panico. The letters throw much light on the personal relations between Wittgenstein and Sraffa but they have almost no philosophical content. An undated letter contains reference to a discussion on what being in a rage looks like, where Sraffa clearly opposed Wittgenstein from a physicalist standpoint, in terms of contraction of facial muscles. This is further evidence in support of the claims in section 2 below.

<sup>4</sup> There are a few more remarks, e.g., (Wittgenstein 1996, 26), which will not be examined here, for reasons stated below.

<sup>5</sup> For reasons unknown to me, the last three words were deleted for the printed version. According to Wolfgang Kienzler, the key word in this passage is “use”, which shows a “pragmatic tendency” in the later Wittgenstein (Kienzler 1997, 53). For my part, I would point out that remarks such as this one open the door to the conception of mathematics as an “anthropological phenomenon” that fascinated the later Wittgenstein. See, e.g., (Wittgenstein 1978, VII, par. 33). However, the key point of this passage is that “the judge does not USE the statute book as a manual of anthropology”: the use of mathematical theorems is thus conceived by Wittgenstein as akin to jurisdiction.

fundamental importance”(Monk 1991, 261). There is a chronological difficulty already noted by Monk: Wittgenstein had already recognized the influence of Sraffa in 1932, while this ‘anthropological’ feature did not emerge until later. At all events, philosophers have diversely interpreted this ‘anthropological’ feature. For example, K. T. Fann claimed that Wittgenstein’s “method of speculative anthropology” – in Fann’s words, the “important method of imagining and constructing simple and complicated ‘language-games’” – is “an adaptation of Sraffa’s method”(Fann 1969, 49-50).<sup>6</sup> Brian McGuinness spoke instead of “the idea of a way of thinking as reflecting the character of a culture” (McGuinness 1982, 39). These suggestive remarks were not, however, accompanied with detailed arguments.<sup>7</sup>

At all events, it is not appropriate to enter here into the intricate exegesis of Wittgenstein’s thought and the *anthropologische Betrachtungsweise*.<sup>8</sup> In the remainder of this section, I shall briefly review an interesting attempt at fleshing out the influence of Sraffa by John Davis, while in the second and third sections, I shall present new suggestions. It should be noted at the outset that, in his preface to *Philosophical Investigations*, Wittgenstein implies that Sraffa helped him (along with F. P. Ramsey)<sup>9</sup> to realize the mistakes he had committed in his *Tractatus*. Furthermore, Wittgenstein already acknowledged (in the passage quoted above) the influence of Sraffa as fundamental in 1932, therefore at a time when he had only begun to move away from the conceptions of his *Tractatus* and when the key ideas of the *Philosophical Investigations* were still in gestation. It is not surprising, therefore, that the anecdote concerning the Neapolitan gesture contains an important critique of an essential feature of the doctrines of the *Tractatus*. In light of these facts, I find it more judicious to try and look for points of contact between Sraffa and Wittgenstein at a stage when Wittgenstein was critically reflecting on his own *Tractatus*, instead of trying to make direct links with ideas that occurred later, such as the notions of ‘language

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<sup>6</sup> On related points, see also (Rhees 1996, 50), (Jacquette 1998, 205).

<sup>7</sup> See, however, (McGuinness 1986) or (Kienzler 1997, 51-55). As for McGuinness, see also the views reported in (Sen 2004, 40), in accordance with the remarks in footnote 5 above.

<sup>8</sup> I shall discuss it in (Marion forthcoming II).

<sup>9</sup> I have discussed the influence of Ramsey on Wittgenstein in numerous places. See (Marion 1995a), (Marion 1998, chap. 4) and (Marion forthcoming I).

games', which first appeared in the *Blue Book* in 1934, although this is neither altogether inappropriate nor impossible.

Furthermore, since there are very few traces of the interactions between the two, one is forced to appeal, when framing suggestions, to background elements in their thinking that bear strong resemblance. I think that is fair to say that Sraffa did not impart a new 'line of thinking' by discussing matters of economic theory with Wittgenstein. However, Sraffa must have reacted to Wittgenstein's ideas from his own standpoint and suggestions of a possible 'influence' must thus appeal to a shared 'background'.

In *Sraffa and the Theory of Prices*, Alessandro Roncaglia was probably the first to come up with parallels between Sraffa's critique of the marginalist theory in the 1920s and Wittgenstein's criticisms of his own *Tractatus*, in order to frame valuable conjectures concerning the influence of Sraffa. He relied on the following parallels between the marginalist theory and the *Tractatus*:

The marginal theory of value and distribution has been constructed, one might say, on the basis of a philosophical position (whether implicit or explicit, conscious or unconscious) similar to that of the early Wittgenstein, e.g., the atomistic basis of theory and reality (the 'economic agents' and the 'goods'), a correspondence between the facts of the world and the elements of the language (that is, the propositions of the theory as a rational description of reality), the claim to a complete description, according to general rules, of everything in the entire world which can be described (the 'general theories' that are so dear to the neoclassics.) (Roncaglia 1978, 122)

Roncaglia thus basically argued from analogy, to the effect that Sraffa's criticisms of the marginalist theory explain similar criticisms by Wittgenstein of similar aspects of the doctrines of his *Tractatus*.<sup>10</sup>

More recently, John Davis has argued in a similar fashion, drawing specific parallels based on Sraffa's 1926 paper 'The Laws of Returns Under Competitive Conditions'. According to Davis, this paper contains a critique of marginalist theory that can be extended to a criticism of a feature of the *Tractatus* already noted by Roncaglia, its 'atomism'. In a nutshell, in neoclassical theory, in particular in the variant by Marshall and Pigou circulating in the 1920s, prices were

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<sup>10</sup> I can only allude to Roncaglia's multi-faceted discussion (Roncaglia 1978, chap. 7). One interesting aspect is his claim that Sraffa may have imparted a strong dislike of 'general' theories that can decide any statement framed in the language of the theory. For criticisms of Roncaglia's discussion of the influence of Sraffa on Wittgenstein, see (Arena 1982).

said to be determined simultaneously and symmetrically in terms of demand and supply and these were in turn conceived as functions. Sraffa wanted instead to show that a commodity's value does not correspond to a "fundamental symmetry existing between the forces of demand and those of supply" (Sraffa 1926,180) and he criticized in his paper the conception of diminishing and increasing returns underlying Marshall's neoclassical supply curve. He believed that in order to obtain a symmetrical treatment of demand and supply, a merging of very different notions of diminishing and increasing returns into a single "law of nonproportional returns" is required (Sraffa 1926,182). Sraffa argued that claims concerning the independence of diminishing and increasing returns with respect to small variations cannot in fact be sustained. Thus determination of commodity values cannot be adequately achieved through partial equilibrium analysis:

If diminishing returns arising from a 'constant factor' are taken into consideration, it becomes necessary to extend the field of investigation so as to examine the conditions of simultaneous equilibrium in numerous industries: a well-known conception, whose complexity, however, prevents it from bearing fruit, at least in the present state of our knowledge, which does not permit of even much simpler schemata being applied to the study of real conditions. If we pass to external economies, we find ourselves confronted by the same obstacle, and there is also the impossibility of confining within statistical conditions the circumstances from which they originate. (Sraffa 1926, 187)

Davis sees a parallel here between Sraffa's criticisms of the neoclassical supply curve and the criticism of Wittgenstein's picture theory, which he expressed through the famous Neapolitan gesture:

[...] in the discussion related by Malcolm, Sraffa had suggested that Wittgenstein's understanding of logical form was mistaken in its reliance upon the autonomous statement as a unit of meaning. A gesture, as a special sort of statement, was not meaningful apart from the social conventions dictating its usage. Similarly, then, in the 1926 article, conventions of usage are mirrored by forms of industry interaction under varying returns, such that just as a statement's meaning is dependent upon social context, so the summary measure of an industry, namely, the commodity value of that industry, cannot be established independently of that industry's interaction with other industries in presence of varying returns. (Davis 1988, 33)

Davis rightly fleshes out, in this passage, the gist of the anecdote of the Neapolitan gesture: statements do not get their meaning solely and directly from sharing a logical form with facts, but also from being embedded within social practices. (This could be construed as the

*anthropologische Betrachtungsweise* imparted by Sraffa.) Surely, the essential affinity to which Davis draws our attention is merely a rejection by both Sraffa and the later Wittgenstein of a highly generic sort of atomism. Furthermore, Wittgenstein's moves away from the *Tractatus* can be related, in his manuscripts, to arguments having of course nothing to do with the marginalist theory. However, it seems right to infer that Sraffa, having precisely rejected the implicit atomism at the basis of the neoclassical theory, could not but be critical of the atomism underlying the doctrines of the *Tractatus*. Given the nature of their work, it is all but natural that Sraffa would argue with Wittgenstein on his own terms. After all, Wittgenstein aimed at producing jargon-free trivialities, with which everyone would agree (Wittgenstein 1953, par. 128). It seems to me therefore correct to argue, as Davis did (using a strategy first laid out by Roncaglia), from motivations in Sraffa's background to his criticisms of Wittgenstein's *Tractatus*. This is essentially what I shall do in the next section.

2. Silently quoting a famous saying by Keynes, Sraffa wrote in his 1926 paper on 'The Laws of Returns Under Competitive Conditions', that, as a result of the marginalist revolution, economic theory had lost

[...] much of its direct bearing upon practical politics, and particularly in regards to doctrines of social changes, which had formerly been conferred upon it by Ricardo and afterwards by Marx, and in opposition to them by the bourgeois thinkers. It has been transformed more and more into 'an apparatus of the mind, a technique of thinking' which does not furnish any 'settled conclusions immediately applicable to policy'. (Sraffa 1926, 180-181)

One can see here that Sraffa perceived the marginalist revolution as a shift in economic theory from problem-solving to a more 'theoretical' science (in the original sense of the Greek *theorein*), a shift that parallels an increase in the use of mathematics. In opposition to this shift, which he perceived as the essence of 'bourgeois economics', Sraffa sought to revive what he saw as the central project of classical political economy:

[...] as a simple way of approaching the problem of competitive value, the old and now obsolete theory which makes it dependent on the cost of production alone appears to hold its ground as the best available. (Sraffa 1926, 187)

One can see here the influence of Sraffa's socialist background on his thinking. As John Eatwell and Carlo Panico wrote: "His socialism demanded an economics that was concrete; that, however abstract, was appropriate to the interpretation of real economic institutions and phenomena" (Eatwell & Panico 1987, 445).<sup>11</sup> Perhaps one should see here the influence of Gramsci.<sup>12</sup>

This was indeed Sraffa's aim from the late 1920s onwards and he is to be credited for providing a new interpretation of the classical economic theory, from William Petty to David Ricardo, in his introduction to Ricardo's *On the Principles of Political Economy and Taxation* (Sraffa 1951) and in his book on *Production of Commodities by Means of Commodities* (hereafter shortened to *Production of Commodities*) (Sraffa 1960). That Sraffa's interpretation is accurate or not may be a matter of controversy but for the purposes of this paper this is beside the point. I shall give a brief sketch, which will be, as usual with such presentations, somewhat lacking in precision.<sup>13</sup> Although this may not appear obvious when reading it, this sketch will provide the elements needed for an understanding of the two points that I shall be making in relation to Wittgenstein. This is in accordance to the strategy presented in the last section.

In the 1920s, the received interpretation of the evolution of economic theory was Marshall's view that "no real breach of continuity" was involved in the move from classical to

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<sup>11</sup> In a conversation with Gilles Dostaler, Sraffa admitted that he could not have written his book had Marx not written *Das Kapital* and that he felt closer to Marx than the "camoufleurs of capitalist reality" (Dostaler 1982, 103). He also said in that conversation that in his book he was describing the same phenomenon as Marx did, namely the class struggle between workers and capitalists; the latter describing it in terms of the appropriation by capitalists of the surplus value, while Sraffa thought that he described it algebraically, when the net product is divided between wages and profit, with the formula  $r = R(1-w)$ , for the profit rate  $r$ , the share of wages in the net product  $w$ , and the Standard ratio or maximum rate of profit  $R$  (Sraffa 1960, 22). The distribution of the physical surplus is precisely what is at stake in the class struggle, according to Sraffa (Dostaler 1982, 103).

<sup>12</sup> Indeed, it is perhaps apposite to see in the perception of the evolution of economic theory away from problem-solving and towards a fully 'theoretical' science (in the sense of *theorein*) as a 'bourgeois' manoeuvre, the influence of Gramsci. Indeed, one could easily redescribe this manoeuvre in Gramscian terms as 'hegemonic'. This is controversial: Nicholas Kaldor has conjectured that Sraffa shifted his interest from money and banking to the classical theory of value as the result of Gramsci's influence (Kaldor 1984, 149), while Alessandro Roncaglia claims that there seems to be no such influence and that Sraffa's research and results should be judged independently from his political background (Roncaglia 1983, 339) (Roncaglia 2000, 11).

marginal theory (Marshall 1952, v); classical economists such as Ricardo were thus portrayed as early demand and supply theorists with somewhat inelastic demand (Marshall 1952, 670-676). Following Sraffa's reconstruction, classical economists were concerned with the laws governing the emerging capitalist economy, which was characterized by a division between workers and land and capital owners, by wage labour as the dominant form of appropriation of other people's work, by a division of labour within and between firms and by co-ordination through interdependent markets, where transactions are mediated through money. Sraffa's approach to the classical economists consisted in recovering their focus on the non-accidental and non-temporary factors governing the economic system. This implied a distinction between 'actual' and 'normal' value, the latter reflecting these invariant features of the system. According to classical economists, in conditions of free competition, the system would gravitate around a 'long-term' position characterized by uniform rates of profit and remuneration for each primary input. One had first to isolate the factors at work to determine income distribution and prices supporting that distribution in a given time and place and only then study the causes, such as capital accumulation or technical change, of changes over time.

There were two important features: first, production was conceived following Petty and the Physiocrats as circular flow, i.e., commodities are seen as produced by commodities, as opposed to the "one-way avenue that leads from 'Factors of production' to 'Consumption goods'" (Sraffa 1960, 93). Secondly, classical economists operated with a concept of 'physical real cost', based upon the underlying assumption that, in the words of Heinz Kurz:

Man cannot create matter, man can only change its form and move it. Production involves destruction, and the real cost of a commodity consists in the commodities destroyed in the course of its production. (Kurz 2004)

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<sup>13</sup> I rely heavily in the following presentation on (Kurz 2004), to which the reader is directed for details.

Therefore the cost of production is calculated ‘objectively’ in terms of the “quantities of things used up”, the surplus being the difference between the quantities produced and the latter. One can argue that the concept of ‘physical real cost’ has its modern origin in William Petty’s *Political Arithmetick* (Petty 1986, 244), and that variants are to be found in François Quesnay, Adam Smith, David Ricardo, James Mill, Robert Torrens, and Karl Marx. Possibly under the influence of Gramsci (Naldi 2000, 92), Sraffa focussed in the late 1920s on distinction between ‘objective’ and ‘subjective’ factors. He profoundly disliked the “subjective, moral point of view” and wanted to do away with ‘subjectivist’ concepts, such as ‘preferences’ or ‘utility’, introduced by the marginalists, and asked for an “objectivisation” ([D3/12/7: 46], quoted in (Kurz & Salvadori forthcoming)), i.e., *reliance on observable, measurable quantities alone, to the exclusion of all ‘subjectivist’ concepts*. Thus no ‘reductionism’ is implied. Indeed, there is no claim that the ‘subjective’ concepts should be reduced to ‘objective’ ones (as in, for example, behaviourism in psychology), only that they should be avoided.

As Sraffa commented in an unpublished note, the ‘objectivist’ point of view is linked with the conception of production as circular:

This point of view implies replacing the notion that “commodities are produced by factors of production” with the other that “commodities are produced by commodities”; the latter amounted to ‘replacing the idea that the process of production has a beginning and an end with that that it has a circular one – an idea first introduced by the *Tableau économique*. ([D3/12/7], quoted in (Kurz & Salvadori forthcoming).)

However, in order to be able to determine the general rate of profits and the exchange ratios of different commodities in terms of the ‘physical real cost’, one would have to *solve a set of simultaneous equations*. The mathematical tools needed to do so were not available to classical economists and they naturally set out to reduce the heterogeneity of commodities to a common

measure. (Hence, for example, Marx’s identification of labour as the common measure of value.)

This is the key to Sraffa’s controversial ‘corn-profit’ interpretation of Ricardo:

The advantage of Ricardo’s method of approach is that, at the cost of considerable simplification, it makes possible an understanding of how the rate of profit is determined without the need of a method for reducing to a common standard a heterogeneous collection of commodities. (Sraffa 1951, xxxii)

As Sraffa noted – this is the basis for *Production of Commodities* –, all that was needed was the solution of a set of simultaneous equations; the problem of ‘common measure’ could be avoided.

To take the simple example of an economy *with no surplus* with three kinds of commodities, tools (*t*), raw materials (*m*) and the food of the labourer (*f*), the productions in three industries can be depicted with the following schema:

$$T_t \oplus M_t \oplus F_t \rightarrow T$$

$$T_m \oplus M_m \oplus F_m \rightarrow M$$

$$T_f \oplus M_f \oplus F_f \rightarrow F$$

Here,  $T_i, M_i,$  and  $F_i$  are the inputs of the three commodities (as means of production *and* means of subsistence) in industry *i* and  $T, M,$  and  $F$  are the total outputs in the three industries; the symbol  $\oplus$  indicates that all inputs on the left-hand side are required to generate the outputs. From this schema, it is easy to figure out the values based on ‘physical real cost’. If one denotes the value of one unit of commodity *i* by  $p_i$ , then one has the following set of equations:

$$T_t p_t + M_t p_m + F_t p_f = T p_t$$

$$T_m p_t + M_m p_m + F_m p_f = M p_m$$

$$T_f p_t + M_f p_m + F_f p_f = F p_f$$

Fixing a standard of value (whose price is equal to unity) allows one to solve the equations. This can be seen using a numerical example (adapted in (Kurz 2003) from an example by Sraffa):

$$2 p_t + 15 p_m + 20 p_f = 17 p_t$$

$$5 p_t + 7 p_m + 4 p_f = 28 p_m$$

$$10 p_t + 6 p_m + 11 p_f = 35 p_f$$

For these equations, the values are:  $p_t = 3 p_m$ ,  $p_m = 2/3 p_f$ , and  $p_f = 1/2 p_t$ . Here, any of the commodities  $t$ ,  $m$ , or  $f$  could serve as ‘common measure’. This sort of ‘algebraic’ thinking – about which more later – is at the heart of Sraffa’s economic thinking: *Production of Commodities* comprises only more complex calculations based on such sets of equations, in order to extend the domain of their applicability, first to cases of production with a surplus and then to joint production. For example, for the determination of the general rate of profits  $r$ , the following set of equations will allow, when a standard of value is fixed, to determine the general rate of profits and prices:

$$(T_t p_t + M_t p_m + F_t p_f) (1+r) = T p_t$$

$$(T_m p_t + M_m p_m + F_m p_f) (1+r) = M p_m$$

$$(T_f p_t + M_f p_m + F_f p_f) (1+r) = F p_f$$

There is no need to go into further details for my present purposes. There are no controversial issues, bearing on, e.g., the case of joint production, that could have any significant effect on what follows. (The validity of the propositions derived by Sraffa or the soundness of his approach is not what is at stake in this paper.) I would like simply to make two observations about the mathematics of *Production of Commodities*.

First, Sraffa limits himself to linear algebra and this should be contrasted with the current use of the point of view of analysis or topology. Since von Neumann, Arrow & Debreu, and McKenzie, use of fixed-point theorem is the most common method in general equilibrium theory. This point will be discussed fully in the next section. Secondly, Sraffa is also careful to provide

algorithms, for example, when proving the uniqueness of the Standard system (chapter V). When Sraffa discusses switching of techniques (chapter XII), he introduces an algorithm for the selection of an optimal set of techniques that has been dubbed recently by Christian Bidard “*l’algorithme de Sraffa*” (Bidard 1998, 785). I shall make brief comments on both algorithms in the next section. For the moment, I would like to point out that it seems to me *to be essential for Sraffa’s whole enterprise that solutions to the sets of equations be at least computable; an ‘existence’ result that merely states that a given set of equations has a solution without giving us any means to compute it would not do*. Indeed, his mathematics are constructive throughout and it seems to me that Sraffa’s viewpoint is that of economics as problem-solving.<sup>14</sup>

My remarks on the influence of Sraffa on Wittgenstein will concern *only* the two fundamental aspects that I have tried to make obvious in the above sketch, namely the ‘objectivist’ standpoint underlying the notion of ‘physical real cost’, and the ‘algebraic’ and ‘algorithmic’ thinking everywhere in appearance in *Production of Commodities*.

My first point is that one of Wittgenstein’s first and most important moves away from the *Tractatus* consists of the adoption, around 1930, of a ‘physicalist’ stance, which is related to Sraffa’s ‘objectivism’. There is evidence for Wittgenstein’s change of mind in passages such as these:

I used to believe that there was the everyday language that we all usually spoke and a primary language that expressed what we really knew, namely phenomena. I also spoke of a first system and a second system. [...] I do not adhere to that conception any more. I think that essentially we have only one language, and that is our everyday language. We need not invent a new language or

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<sup>14</sup> One can trace the origin of this point of view in R. M. Goodwin’s interpretation of *tatônement* as an algorithm in (Goodwin 1951). But it could also serve to characterize the classical tradition, while it is somewhat lost in the neoclassical tradition, where the approach is of a more *formalist* nature. There is an obvious link between viewing economic theory as *problem-solving* and insisting on *constructive content*, that has been expressed recently by Kumaraswamy Velupillai: “standard mathematical economics is replete with existential theorems without the slightest concern over their constructive or algorithmic status. [...] It is also why economic theory has not been a pleasant playing field for those of us who would like to interpret the cardinal aim of the subject to be *problem-solving*” (Velupillai 2000, 181).

construct a new symbolism, but our everyday language is *the* language, provided we rid it of the obscurities that lie hidden in it. (Wittgenstein 1979, 45-46)

I do not now have phenomenological language, or ‘primary language’ as I used to call it, in mind as a goal. I no longer hold it to be necessary. All that is possible and necessary is to separate what is essential from what is inessential in *our* language. [...] A recognition of what is essential and what is inessential in our language if it is to represent, a recognition of which parts of our language are wheels turning idly, amounts to the construction of a phenomenological language. (Wittgenstein 1975, § 1)

In a nutshell, in the *Tractatus*, Wittgenstein discussed the possibility of an analysis of propositions of “everyday language” in terms of their constituents. While this is not clearly stated, he conceived these propositions as using terms referring to physical, observable objects, while he believed that the process of analysis must terminate at a level of ‘elementary propositions’ that are a concatenation of ‘simple names’, which refer directly to ‘simple objects’. Because of the brevity of the text, Wittgenstein left many questions unanswered and the nature of these objects remains controversial. It can be argued that, while they are not analogous to Russell’s ‘sense-data’, they have at least this in common with Russell’s ‘objects of acquaintance’ that they have to be given to me in immediate experience.<sup>15</sup> What is described in the above quotations as the abandonment of “phenomenological language, or ‘primary language’” is the result of Wittgenstein’s realisation, in 1929, that there is no such thing as a ground level constituted of ‘elementary’ propositions about ‘simple objects’ given to me in experience. This realisation forced Wittgenstein to throw away the very idea of an ‘analysis’: “We need not invent a new language or construct a new symbolism”.

To see how this change could be related to Sraffa’s ‘objectivism’, one must first clarify the meaning of the term ‘physicalism’. There was at that time a debate among the members of the Vienna Circle, in the early 1930s, about the adoption of a ‘physicalist’ language as the universal

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<sup>15</sup> I have provided arguments in (Marion 2004, 79-84).

language of science, a thesis that involves the rejection of methodological dualism. The term was, however, ambiguously defined by the proponents of ‘physicalism’, Otto Neurath and Rudolf Carnap. Indeed, Neurath, who was also strongly influenced by Marx and responsible for the official economic views of the Vienna Circle, presented at times the universal language as simply the “language of physics” (Neurath 1983, 54-55).<sup>16</sup> But a language that would contain only metrical concepts would not be suited for the job and Carnap weakened the physicalist thesis and developed a ‘thing-language’, which would contain also qualitative concepts provided that they “refer to observable properties of things and observable relations between things” (Stegmüller 1969, 293).

Opposed to this is the language based on the “stream of experience” of a single, solitary subject, the language of ‘autopsychological objects’, to use an expression taken from Carnap’s *Logical Structure of the World* (Carnap 1967, par. 58). This language is related to Ernst Mach’s ‘phenomenalism’, i.e., to his attempt at reconstructing science on the basis of atomistic, ‘simple sensations’ (Mach 1959). While Carnap had remained neutral in *Logical Structure of the World* with respect to the choice of the basic language, in an article from 1931, ‘Die physikalische Sprache als Universalsprache der Wissenschaft’, he decided in favour of the physicalist stance already adopted by Neurath (Carnap 1995). In short, Neurath and Carnap chose to adopt of physicalist language because they believed that the adoption of the ‘phenomenalist’ language would lead towards solipsism, i.e. the unpalatable thesis that only one mind exists.

There is no evidence that Sraffa was aware of these developments within the Vienna Circle. However, in 1928 he had already read and annotated carefully A. N. Whitehead’s *Science and the*

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<sup>16</sup> To complicate matters, Neurath also spoke of the unified language as a purified version of everyday language, which he identified with the language of physics (Neurath 1983, 62 & 91).

*Modern World* (Whitehead 1925).<sup>17</sup> In this book, Whitehead described modern science in terms reminiscent of the ‘physicalist’ language of the Viennese:

Science was becoming, and has remained, primarily quantitative. Search for measurable elements among your phenomena, and then search for relations between these measures of physical quantities. (Whitehead 1925, 66)

Whitehead also described his own standpoint as an “objectivist philosophy adapted to the requirement of science and to the concrete experience of mankind” (Whitehead 1925,129) and he further described his ‘objectivism’ in these terms:

This creed is that the actual elements perceived by our senses are *in themselves* the elements of a common world; and that this world is a complex of things, including our acts of cognition, but transcending them. According to this point of view the things experienced are to be distinguished from our knowledge of them. (Whitehead 1925, 128-129).

This last sentence is meant to contrast ‘objectivism’ with ‘subjectivism’, i.e., the view that “what is perceived is not a partial vision of a complex of things generally independent of that act of vision” (Whitehead 1928, 128). What Whitehead calls here ‘subjectivism’, one usually calls ‘idealism’. In Sraffa’s time, Marxists were critical of ‘idealism’, which they perceived as bourgeois philosophy; one should recall here Lenin’s criticisms of Ernst Mach’s ‘phenomenalism’ in *Materialism and Empirio-Criticism* (Lenin 1970) or Gramsci’s critique of Croce in his *Quaderni del carcere*.<sup>18</sup> It seems natural for the likes of Carnap, Neurath and Sraffa to adopt an ‘objectivist’ stance. On the other hand, the ‘phenomenalist’ language of the Viennese bears strong resemblance to the ‘subjectivism’ described by Whitehead and rejected by Sraffa. This

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<sup>17</sup> I would like to thank Heinz Kurz for having pointed this out to me.

<sup>18</sup> Lenin’s critique of Mach was clearly off the mark. The motives for Gramsci’s call for an “anti-Croce” (Gramsci 1971, 371) are complex and the attack on his idealism only part of it. For Gramsci’s discussion of ‘subjectivism’ and ‘objectivism’, see (Gramsci 1971, 440-448). He did not simply reject ‘subjectivism’ but called for a synthesis (Gramsci 1971, 402). In this, he is not followed by Sraffa.

‘subjectivism’ is, however, the basis to Friederich von Hayek’s thinking about economics.<sup>19</sup>  
The battle lines are here clearly demarcated.

One should now note that in the passages quoted above, Wittgenstein’s alternatives are between having or not having a ‘basic’, ‘primary’ language that expresses “what we really knew”, and not, as was the case within the Vienna Circle, a choice between the ‘thing language’ or the language of the ‘autopsychological’. It is also an alternative between ‘everyday’ language and a so-called ‘phenomenological’ language. But Wittgenstein’s ‘everyday’ language sounds very much like Carnap’s ‘thing language’:

[...] our ordinary language [speaks] of the events in our environment by talking of objects (things, bodies), ascribing properties to them, or relating them to each other, etc. (Wittgenstein 1979, 254)

As for Wittgenstein’s use of the term ‘phenomenology’, it is derived from that of physicists such as Ernst Mach or Ludwig Boltzmann, whose works were known to him. In physics, the idea of a phenomenology has nothing to do with a contrast between entities such as ‘ideas’, ‘sense-impressions’, or ‘sense-data’ and ‘material objects’, as is the case with the various philosophical forms of ‘phenomenalism’ such as Mach’s or Russell’s; it has to do with the possibility of a self-sufficient description of experience, e.g., a thermodynamics that does not assume molecules.<sup>20</sup> The idea that there could be such a description for ordinary language was at the basis of the *Tractatus*, i.e., the idea of a ‘primary’ language – the distinction between ‘primary’ and ‘secondary’ language is taken from Hertz’s *Principles of Mechanics* (Hertz 1899), another book read by Sraffa – which would express “what we really knew”. But this ‘phenomenological’ language would be a pure description of immediate experience or the ‘stream

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<sup>19</sup> Hayek’s first book, *The Sensory Order*, was an exercise in anti-‘objectivism’ (Hayek 1952). See (Nadeau 2001) for a discussion.

<sup>20</sup> See, e.g., (Boltzmann 1974, 93f.). This is the ‘phenomenological thermodynamics’ mentioned by von Neumann in (von Neumann 1945, 1).

of experience', which is nothing more than Carnap's 'autopsychological' basis (Carnap 1967, par. 64). This is the idea that Wittgenstein abandoned in 1929.

In the development of Wittgenstein's thought, the move from 'phenomenological' to 'physicalist' language is neither the sole nor the most important step away from the *Tractatus*. Its importance for later developments is a matter of controversy. Merrill and Jaakko Hintikka argued that it is related to the two major arguments of the later philosophy, on 'private language' and 'rule-following' (Hintikka & Hintikka 1986, 138). Again, the validity of such claims is not at stake here. One simply cannot deny that Wittgenstein change his mind about the need for a phenomenological language. In absence of any direct evidence linking Wittgenstein's change of mind to discussions with Sraffa it is not possible to draw any firm conclusions. However, if it were the case that Sraffa criticized the atomism of the *Tractatus* because his own thinking in economic matters was anti-atomistic, it would hardly be possible for him not to have criticized in conversation the phenomenological stance of the *Tractatus*, since it amounts to a form of subjectivism which he so staunchly rejected in his own thinking about economic matters. The parallel between Sraffa and Wittgenstein is here too very striking.

**3.** My second suggestion is that the mathematics used in *Production of Commodities* and Sraffa's 'algorithmic' thinking are in conformity with Wittgenstein's constructivist stance on the foundations of mathematics. It is even less appropriate here to speak of influence because one can claim that the *Tractatus* already embodies a constructivist philosophy of mathematics, so talk of a change of mind initiated by Sraffa's criticisms is out of question. Moreover, Sraffa had little mathematical training and knowledge; while writing *Production of Commodities*, he repeatedly needed help from the (Cambridge) mathematicians cited in the preface, Abram Besicovitch, Frank

Ramsey and Alister Watson (Sraffa 1960, vi-vii).<sup>21</sup> For example, the (algorithmic) proof of the existence of a Standard Commodity in note (D3/12/39-42) is literally from Besicovitch's hand.<sup>22</sup> Furthermore, he never had any ostensible interest in issues concerning the foundations of mathematics. Therefore, *he could hardly be suspected of having had strong convictions in these matters*. (And strong convictions are what is needed to adopt a minority view such as constructivism in mathematics.) On the other hand, these are matters to which Wittgenstein devoted a great deal of space.

At any rate, even if Sraffa's attitude towards mathematical methods in economics is not derived from some prior philosophical belief about what is good mathematics or not, it remains that the mathematics used in *Production of Commodities* are correctly characterized as "constructivist", while the true reasons why Sraffa eschewed non-constructive mathematics are surprisingly in line with Wittgenstein's own thoughts about mathematics. The claim that Sraffa implicitly adopted a constructivist stance is at any rate not new, having previously been argued for by numerous economists (Bidard 1998), (Chakravarty 1989), (Dore 1989), (Punzo 1986, 1989, 1991), (Schmidt 1985, 1988, 1990).<sup>23</sup> My discussion will differ from these because of the new connections that I establish with Wittgenstein's ideas. Before doing so, however, I shall give a very elementary and very brief presentation of the constructivist critique of 'existence' theorems in mathematics and discuss in more details the contrasts between Sraffa's mathematics and von Neumann's celebrated 1937 result on the existence of an economic equilibrium, as well as the differences between their respective models.

There can be no entirely satisfactory definition of constructivism in mathematics, since the

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<sup>21</sup> On this, see (Kurz & Salvadori 2001, 2004b).

<sup>22</sup> I would like to thank Neri Salvadori for pointing out this note for me.

term refers to half a dozen programmes or schools of thought that have little in common.<sup>24</sup> However, these schools have at least one point in common, the rejection of pure ‘existence’ theorems. In simple terms, ‘existence’ theorems are statements of the form  $\exists x A(x)$  that merely claim there exists an object  $x$  possesses the property  $A$ , while the proof does not provide the means to exhibit an instance that has the property  $A$ . For constructivists, one must be able to exhibit, with a construction, such an instance. There are some philosophical grounds for such claims. ‘Existence’ theorems can be justified by appeal to a ‘Platonist’ philosophy about a realm of abstract entities existing independently of our cognitive capacities, while the constructivist view derives from a more ‘Kantian’ approach: mathematical entities are constructed by us. At all events, there are some straightforward logical consequences. For example, this means that, if a statement  $A(x)$  does not hold for every integer  $x$ , one is not allowed automatically to infer that there must exist an integer  $x$  for which this statement is false. In other words, constructivists prohibit use of this logical principle:

$$\neg \forall x A(x) \rightarrow \exists x \neg A(x).$$

This prohibition originates in the 19<sup>th</sup> century, in the writings and teaching of Leopold Kronecker, and it was developed in the 20<sup>th</sup> century by L. E. J. Brouwer into a full-blown critique of the Law of Excluded Middle:

$$A(x) \vee \neg A(x)$$

Brouwer’s critique is at the origin of the school of intuitionism, which is a better-known variant of constructivism. Among other features, intuitionistic logic distinguishes itself by the rejection of the principle of double negation elimination:

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<sup>23</sup> This opinion is, of course, not universally shared. For sceptical comments, see (Schefold 2004, 317). Heinz Kurz and Neri Salvadori have also expressed their scepticism to me in private correspondence.

<sup>24</sup> For basic information on the various schools of constructivism in mathematics, the reader should consult

$$\neg\neg A(x) \rightarrow A(x).$$

‘Existence’ proofs are often obtained by use of this principle in a *reductio ad absurdum*: assuming  $\neg A$  implies a contradiction so it must be the case that  $A$ . This principle of proofs is thus rejected by constructivists.

It is often claimed that adoption of intuitionistic logic would entail giving up mathematical results. That this viewpoint is the appropriate one for the foundations of mathematics is quite another question. However, one should not think that it is often (but in principle not always) the case that ‘existence’ proofs only hide ignorance of an algorithm and that constructive proofs can be supplied. For example, Errett Bishop has given a constructive version of large parts of classical analysis in *Foundations of Constructive Analysis* (Bishop 1967). This fact will be become important below.

L. E. J. Brouwer also happens to be one of the founders of modern topology but he did not adhere to his own logical strictures in his work on topology (this does not mean that it has no constructive content) and his fixed-point theorem is a perfect example of an ‘existence’ proof. This example is worth mentioning since John von Neumann’s celebrated result in ‘A Model of General Economic Equilibrium’ that *there exists* a set of prices for which supply equals demand in every market in any set of equations used to describe a competitive economy is also an ‘existence’ theorem (von Neumann 1945). It asserts the existence of a certain object, the set of prices, without giving the means (algorithm) to obtain it. The argument is of the form: the non-existence of a solution involves a contradiction, therefore it is impossible that there be no solution.

As one can see from the original title of the paper, ‘Über ein Ökonomisches

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(Troelstra and van Dalen 1988) or (Bridges & Richman 1987).

Gleichungssystem und eine Verallgemeinerung des Brouwerschen Fixpunktsatzes' (von Neumann 1937), the proof contains a lemma using Brouwer's fixed-point theorem. A generalization of the latter by Kakutani (Kakutani 1941), stands at the basis of much of today's general equilibrium theory, where the commodity space has the structure of a real vector space (with useful properties such as convexity). By contrast, the mathematics involved in *Production of Commodities* are limited to the perfectly constructive sphere of linear algebra.<sup>25</sup> Sraffa's model can be given a matrix formulation and the key proof of the book, i.e., the existence of a unique Standard system, can be obtained using purely algebraic means, i.e., the Perron-Frobenius theorem on non-negative matrices.<sup>26</sup> However, one should note that Sraffa avoided a matrix formulation. (The reasons for this will be discussed below.) Two preliminary remarks concerning this contrast are in order.

First, merely highlighting the difference between the constructive setting for Sraffa's model and the non-constructive nature of von Neumann's result, may bring confusion. Since the famous exchange between Kaldor and Solow at the Corfu conference on 'Capital Theory' in 1959, the interpretation of von Neumann's model has been a matter of controversy. In short, von Neumann's growth model appears at a juncture. It is hailed by neoclassical economists as a crucial step between Cassell and Debreu, while others emphasize features of the model that justify looking at it, on a par with Sraffa's model, as part of the tradition of classical economics. Somewhat caught in the middle are some modifications or extensions of von Neumann's model that retain features from both traditions, such as Brody's (Brody 1970), Goodwin's (Goodwin & Punzo 1987) or Pasinetti's (Pasinetti 1981). It is of course not the purpose of this paper to

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<sup>25</sup> For a non-set-theoretical introduction to linear algebra that makes patent its constructive content, see (Edwards 1995).

adjudicate this debate, although the arguments towards a rapprochement of von Neumann's model with the classical tradition are more compelling (Kurz & Salvadori 1993, 2004a). In this context, emphasis on the constructive character of the mathematics underpinning Sraffa's model might provide grounds for the belief that, while Sraffa's model is clearly of 'classical' inspiration, von Neumann's belongs to the neoclassical tradition. What is at stake in this debate is not the mathematics but the economic features of the model (e.g., that it is a 'long-period' model or not) and, on that score, there are good grounds to link von Neumann's model with the classical tradition. The issue about mathematics cuts across this debate for the simple reason that the use of non-constructive mathematics is not a necessary and sufficient condition to belong to the neoclassical tradition. As Punzo pointed out, Cassel was aiming at a constructive proof of the existence of an equilibrium (Punzo 1989, 42). Chakravarty has also insisted on the fact that the non-constructive mathematics in von Neumann's paper are not sufficient in themselves for characterizing his model as neoclassical (Chakravarty 1989, 73). However, von Neumann's paper as opened the door to a more 'formalist' mathematical treatment of economic questions which is undoubtedly the mark of neoclassical mathematics. I shall come back to this important point.

Secondly, one should note that von Neumann's growth model in can be seen as an interpretation of his saddle-point or 'minimax' theorem (Dore 1989, 87-92), which is also proved with help of the fixed-point theorem (von Neumann 1928). As pointed out earlier, most 'existence' theorems are *bona fide* and constructive proofs can be obtained. A proof by Ville had already shown von Neumann's use of the fixed-point theorem to be redundant (Ville 1938). It has been shown also that the latter's methods were sufficient to obtain von Neumann's equilibrium result (Georgescu-Roegen 1951). But these results still appeal to topological tools, while George

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<sup>26</sup> See (Burmeister 1968). For an overview of the role of the Perron-Frobenius theorem, see (Punzo & Velupillai

Dantzig, who is better known for the invention of the simplex algorithm, has given a constructive proof of the minimax theorem (Dantzig 1956), while a constructive proof by linear programming methods has also been given for solutions in the generalized von Neumann model (Kemeny et al. 1956). These results show that, from a constructivist standpoint, uses of the fixed-point theorem in economy by von Neumann are essentially harmless, since the results can also be obtained constructively.

This point serves to reinforce my comment, above, concerning the fact that a discussion of the mathematics underpinning Sraffa's and von Neumann's models has precious little bearing on the issue of the interpretation of the latter as pertaining or no to the classical tradition. There has been a recent tendency to picture Sraffa's *Production of Commodities* and Debreu's *Theory of Value* as alternative explanations against a common background of perfect competition in market economy, while comparisons were made between Sraffa's and von Neumann's models (Steedman 1976) and (Schefold 1978, 1980). It is not clear that minimizing his interest in conditions of imperfect competition provides an adequate picture of Sraffa's overall intellectual project.<sup>27</sup> Be this as it may, it seems to me that the use, in the context of the problem of the selection of an optimal set of techniques, of non-constructive methods derived from von Neumann's famous paper (von Neumann 1945),<sup>28</sup> is less faithful to the spirit of Sraffa's *Production of Commodities* than, say, Christian Bidard's approach in 'An Algorithmic Theory of the Choice of Techniques' (Bidard 1990). However, in this context the use of non-constructive methods is harmless, as Bidard himself recognized (Bidard 1998, 784-787). The real difficulty comes with the fact that von Neumann's 1937 result opened the door to a more 'formalist' approach to the use of

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1984).

<sup>27</sup> See (Arena 1998).

<sup>28</sup> For example, in e.g. in (Schefold 1980) and (Salvadori 1984).

mathematical methods in economics.<sup>29</sup>

Christian Schmidt has argued, using a semantical characterization *à la* Carnap of the relation between mathematical formalism and economic reality, for the contrast between Sraffa's constructivism, for which it is impossible to disconnect the theory from its interpretation, and *inter alia* the formalist approach of neoclassical economists such as Debreu, for whom "the theory, in the strict sense, is logically entirely disconnected from its interpretations" (Debreu 1959, viii).<sup>30</sup> (There are difficulties, however, with Schmidt's attempt at giving a more precise characterization of Sraffa's implicit constructivism, to which I shall come back.) Indeed, Sraffa was clearly sensitive to the fact that the concepts he used and the relationships he discussed had to have a clear, straightforward economic interpretation; *this is, after all, what his 'objectivism' is all about*. His collaboration with the mathematicians (Besicovitch, Ramsey and Watson), shows that Sraffa was always reasoning first in economic terms and that he would seek help from mathematicians only at a later stage to complete the proofs. An entry in his diary from May 31, 1958 is rather telling:

Besicovitch insists that I publish: the fact that I was able to foresee interesting mathematical results shows that there must be something in the theory. (Quoted in (Kurz & Salvadori 2004b, 259))

It may be that Sraffa's avoidance of the matrix formulation is to be explained by his wish to avoid heavy reliance on mathematics in his exposition. It remains that the establishment of a one-to-one correspondence, i.e., an isomorphism between the mathematical entities and the real entities was fundamental to Sraffa and the idea that there might be more than one interpretation of the model or that the "theory [...] is logically entirely disconnected from its (economic)

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<sup>29</sup> Lionello Punzo has investigated the motives for von Neumann's use of topological methods in his dealings with Karl Menger's *Mathematical Colloquium* in Vienna and in the 'formalist' philosophy of Hilbert, by whom he was much influenced in the late 1920s (Punzo 1989, 1991).

interpretations” are simply foreign to him.<sup>31</sup> Moreover, uses of the fixed-point theorem and the topological standpoint within the neoclassical tradition raise problems with the one-to-one correspondence, since completed infinities are introduced that have no empirical counterpart. And the matrix formulation implies constant returns to scale (Dore 1989, 84), an assumption that Sraffa did not wish to make (Sraffa 1960, v).<sup>32</sup> As Mohammed Dore pointed out, such difficulties are no doubt reasons why Sraffa refrained from using these methods:

Sraffa does not express his results by using matrix methods. A methodological purist, and philosophically inclined towards constructivism, he could not possibly compromise his inquiry by embedding his theory in a more general mathematical structure which could raise the possibility of additional assumptions, a baggage that he did not need. For example, expressing the Sraffa system by matrix methods would imply that he was assuming constant returns to scale, an assumption he did not wish to make. [...] embedding entails the potential of a loss of focus, unless the one-to-one correspondence between mathematical objects and economic entities is carefully preserved, with no loss of economic intuition. (Dore 1989, 87)

All this is fine, except perhaps the claim that Sraffa was “philosophically inclined towards constructivism”, a claim for which there is no direct evidence. This is where Wittgenstein’s philosophy of mathematics, properly understood, may help clarifying matters.

It can easily be shown that Wittgenstein shared with other constructivists their rejection of ‘existence’ proofs.<sup>33</sup> There is only one known mathematical proof due to Wittgenstein and it is a constructive version of a well-known ‘existence’ proof by Euler of the infinity of prime numbers (Wittgenstein 1994, 325).<sup>34</sup> But Wittgenstein criticized some of the arguments of Brouwer and the intuitionists (thus causing quite a lot of confusion) and he reasoned from different

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<sup>30</sup> See (Schmidt 1988, *passim*) and (Schmidt 1990, 105-107).

<sup>31</sup> Joan Robinson famously wrote about *Production of Commodities* as a ‘prelude’ to a critique of the marginalist theory that “the function of pure logic is to liberate us from nonsense, not to tell us what we ought to believe” (Robinson 1965, 34). This reading seems to me to be inaccurate for the very reason that Sraffa never conceived his model as disconnected from its interpretation.

<sup>32</sup> Even though he did make some assumptions on returns. See (Salvadori 1985).

<sup>33</sup> For example, (Wittgenstein 1978, v, §§ 25 & 46, vii, § 41).

premises.<sup>35</sup> The fundamental premise to Wittgenstein's philosophy of mathematics is that mathematics is essentially 'algorithmic', i.e., a sort of high-level abacus activity. According to him, mathematical statements are merely *Scheinsätze*: they are not pictures, they do not express a thought, like ordinary propositions (Wittgenstein 1922, par. 6.2). Therefore, when mathematics is applied to a real-life situation to solve a problem, it cannot be conceived as a 'description', let alone as an 'explanation'. It merely is a calculation that allows one to make a transition from one ordinary proposition to another. Wittgenstein expressed this with great clarity in the *Tractatus*:

In life it is never a mathematical proposition which we need, but we use mathematical propositions *only* in order to infer propositions which do not belong to mathematics to others which equally do not belong to mathematics. (Wittgenstein 1922, 6.211)

Thus, for Wittgenstein:

Mathematics is always a machine, a calculus. The calculus does not describe anything. (Wittgenstein 1979, 106).<sup>36</sup>

The view is also expressed in a passage dating from 1931:

Mathematics consists entirely of calculations. In mathematics *everything* is algorithm and *nothing* is meaning; even when it doesn't look like that because we seem to be using *words* to talk *about* mathematical things. Even these words are used to construct an algorithm. (Wittgenstein 1974, 468 & 1994, 321)

If mathematics is conceived as a sort of high-level abacus, then it is clear that there is no room for 'existence' proofs, where an algorithm is lacking but also where the  $\exists x A(x)$  would be a 'description'. Indeed, Wittgenstein abhorred above all the 'philosophical' view that a mathematical statement  $\exists x A(x)$  could be a 'description' on a par with an ordinary statement that describes reality.

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<sup>34</sup> There is a detailed analysis of that proof and the light it sheds on Wittgenstein's views on mathematics in (Mancosu & Marion 2003).

<sup>35</sup> I have offered my interpretation of Wittgenstein on mathematics in (Marion 1998).

<sup>36</sup> This fundamental feature of Wittgenstein's philosophy of mathematics is in line with Kronecker's 'philosophy'. See (Marion 1995b).

One consequence of Wittgenstein's refusal to conceive mathematical statements as 'descriptions' on a par with empirical statements is his refusal to recognize that there is anything like a *metamathematics*, i.e., mathematical statements about mathematical statements. This led him to a straightforward rejection of Hilbert's metamathematics:

The system of calculating with letters is a new calculus; but it does not relate to ordinary calculation with numbers as a metacalculus does to a calculus. *Calculation with letters is not a theory*. This is the essential point. In so far as the 'theory' of chess studies the impossibility of certain positions it resembles algebra in its relation to calculation with numbers. Similarly, Hilbert's metamathematics must turn out to be mathematics in disguise. (Wittgenstein 1979, 136).

Again, all this is highly controversial, but the correctness of these views is not what is at stake here. I wish to attract attention to the fact that the 'algebraic' and 'algorithmic' thinking at the heart of *Production of Commodities* (as presented in the previous section) fits very well Wittgenstein's conceptions. The above discussion of the anti-formalist aspect of Sraffa economic thinking is also in perfect conformity with Wittgenstein's criticisms of the formalist philosophy of Hilbert. This also fits the demands of Sraffa's socialist convictions, alluded to at the beginning of that section. Moreover, a very interesting parallel obtains between Wittgenstein's life-long critique of the view that mathematics is essentially an activity of description of a world of abstract objects, as opposed to the provision of algorithms that solve practical problems, and Sraffa's belief, already quoted, that economic theory "has been transformed more and more into 'an apparatus of the mind, a technique of thinking' which does not furnish any 'settled conclusions immediately applicable to policy'" (Sraffa 1926, 181). For both Sraffa and Wittgenstein, mathematics provides us with algorithms that allow us "to infer propositions which do not belong to mathematics to others which equally do not belong to mathematics", not with

propositions which describe a (idealized economic) reality.<sup>37</sup>

It appears, however, that Sraffa had a less than perfect grasp of these issues, as the following story shows. In section 37 of *Production of Commodities*, Sraffa presented, as a proof of the uniqueness of the Standard System, an algorithm which consists of the repetition of two steps (“changing the proportions of the industries” and “reducing in the same ratio the quantities produced by all industries, while leaving unchanged the quantities produced by all industries” (Sraffa 1960, 26)), until a solution is found.<sup>38</sup> As Heinz Kurz and Neri Salvadori have shown in their careful analysis of Sraffa’s papers, Alister Watson,<sup>39</sup> who visited Sraffa to help, had qualms about the first step: there are infinitely many ways to perform it but steps in an algorithm need to be uniquely defined. As a mathematician, Watson raised the point but, as Kurz and Salvadori note, “it is not clear whether Sraffa shared Watson’s concern” (Kurz & Salvadori 2001, 273).

I would like to conclude this section with a few critical remarks concerning Schmidt’s interpretation of Sraffa’s constructivist stance. Taking his lead, *inter alia*, from the fact that Sraffa’s algebraic tools are constructive (Schmidt 1990, 110), he claimed that Sraffa’s stance on the foundations of economics is “the only known example of a treatment of theoretical economics directly in accordance with the principles of [Brouwer’s] intuitionism” in the foundations of

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<sup>37</sup> One should note here Maurice Dobb quoting with approval par. 6.211 of the *Tractatus* in *Theories of Value and Distribution since Adam Smith* (Dobb 1973, 8). This shows awareness and appreciation in Sraffa’s entourage of Wittgenstein’s standpoint on mathematics.

<sup>38</sup> For a formal version of the algorithm, see (Kurz & Salvadori 2001, 272-273).

<sup>39</sup> Little is known about A. G. D. Watson (1908-1982). He was a scholar and Fellow of King’s and an Apostle. He is said to have introduced Wittgenstein to Turing. A mathematician, he published only one philosophy paper on ‘Mathematics and its Foundations’ in *Mind* (Watson 1938), in which he expressed views akin to Wittgenstein’s. According to Georg Henrik von Wright (private communication) Wittgenstein recommended reading that paper. During the war, Watson worked for the Admiralty, first on radio communication, then sonar. He finished his career working on oceanography. Watson was also known for his communist sympathies prior to the war. (It was claimed after his death that he might have been a spy, linked to the famous Cambridge ring.) It is only after the war that Watson travelled to Cambridge to help Sraffa.

mathematics (Schmidt 1988, 17).<sup>40</sup> This labelling, taken literally, is incorrect. Brouwer's intuitionism distinguishes itself from other constructivist schools by some idiosyncrasies.<sup>41</sup> The most obvious one is Brouwer's claim that mathematics is the product of the mental activity of a solipsist mathematician, whose results are only imperfectly communicated to others through language; in particular, the fundamental sequence of natural numbers is based on the mathematician's intuition of time. This arch-subjectivist viewpoint could hardly be said to be congenial to Sraffa or, for that matter, to Wittgenstein, who thought that this is "all bosh" (Wittgenstein 1976, 237).<sup>42</sup> It is true that, in opposition to the Arrow-Debreu model, which leaves no room for time (e.g., all transactions are assumed to take place at the present instant, the equilibrium consists of a simultaneous clearing of all markets, etc.) Sraffa's, like von Neumann's, is a long-period model. It seems to me inappropriate, however, to draw analogies between Sraffa's key notion of 'self-replacing state' and intuitionist notions such as 'infinitely proceeding sequences' or 'law-like sequences' (Schmidt 1990, 107 & 109). These are fundamentally different notions.

At any rate, it turns out that Schmidt's considered judgement is that Sraffa went beyond intuitionism: he conjectures that there is an intimate connection with the peculiar brand of constructivism presented by Wittgenstein in his *Remarks on the Foundations of Mathematics* (Wittgenstein 1978). Unfortunately, Schmidt makes a few mistakes, at this stage, in trying to substantiate his claim (Schmidt 1990, 112). The implication here is that Wittgenstein's stance on the foundations of mathematics was of a more radically constructivist nature than intuitionism, a sort called 'strict finitism'. Once more, this labelling seems inappropriate. Strict finitism can be

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<sup>40</sup> See also (Schmidt 1985, 57f.) and (Schmidt 1990, 107f.).

<sup>41</sup> See the papers collected in vol. 1 of (Brouwer 1975) and (Bridges & Richman 1987) for the differences between intuitionism and other constructivist schools.

defined in terms of a distinction between possibility *in principle* and possibility in practice or *feasibility*. To see what is meant here, it suffices to pick an arithmetical operation such as exponentiation and a number such as  $2^{63553}$ . This number consists of two decimal numbers whose position shows what operation must be performed on them (multiply 2 by itself 63553 times) to obtain the number in the decimal system. It is usually assumed that although it is impossible *in practice* to carry out this task – the universe does not contain enough matter to carry out the computation – and thus to obtain the number in the decimal system which corresponds to  $2^{63553}$ , the task can be achieved *in principle*. Strict finitism is defined by the rejection of everything which is outside of the domain of the ‘feasible’.<sup>43</sup> This point of view may sound too radical, but nowadays the concept of ‘feasibility’ has gained renewed importance in theoretical computer science (where the complexity of algorithms is of fundamental importance) under the name of ‘polynomial-time computability’ – a stricter notion than mere ‘effective’ computability.<sup>44</sup> Schmidt assumes that Wittgenstein was a strict finitist but this interpretation is highly controversial.<sup>45</sup> Furthermore, as the above already made clear, there is no trace of any preoccupation about the complexity of algorithms in Sraffa.<sup>46</sup> This shows, once more, the limits of Sraffa’s thinking about these matters.

4. Wittgenstein’s thinking in the early 1930s is characterized by his abandonment of the search for a ‘phenomenological’ language in favour of an analysis of ordinary language, conceived broadly

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<sup>42</sup> On the rather complex philosophical relations between Brouwer and Wittgenstein, see (Marion 2003).

<sup>43</sup> Obviously, the issues are more complex. For a more detailed presentation, see (Gandy 1982).

<sup>44</sup> Some economists, e.g., (Velupillai 2000), are now taking computability and polynomial-time computability seriously.

<sup>45</sup> See (Marion 1998a, chap. 8) for a refutation.

<sup>46</sup> In Sraffa’s background, I could only find such preoccupation in Enrico Barone’s ‘The Ministry of Production in the Collectivist State’ (Barone 1967).

as ‘physicalist’ and by a working out of the constructivist stance on mathematics already implicit in the *Tractatus*. Wittgenstein’s remarks about his conversations with Sraffa indicate that he gained from them an ‘anthropological’ way of looking at things. Apart from some brief remarks, I have stayed away from a discussion of the meaning of this *Betrachtungsweise*. I have tried instead to contribute to our understanding of the relation between Wittgenstein’s thought and Sraffa’s by concentrating on these two aspects, physicalism and constructivism, that have been for the most part overlooked in the secondary literature. In both cases, it seems to me that remarkable parallels obtain between the thoughts of these two Trinity men.<sup>47</sup>

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<sup>47</sup> An ancestor to this paper, which remains unpublished, was written jointly with Don Ross, for which we had occasion to thank Gilles Dostaler and Ken Hugues. An earlier version of the present paper, for which I am solely responsible, was delivered at a meeting on ‘Piero Sraffa (1898-1983)’, at the *Istituto Italiano di Studi Filosofici*, Naples, in October 2003. At that meeting, Heinz Kurz gave a presentation based on his paper with Neri Salvadori, ‘“Man from the Moon”. On Sraffa’s Objectivism’ and Kevin Mulligan, who was in the audience, pointed out to me the connection with Wittgenstein’s physicalism. For this, he is warmly thanked. I would like also to thank for their comments and help, Brian McGuinness, Heinz Kurz, Carlo Panico, Lionello Punzo, Neri Salvadori and Jonathan Smith, the Archivist at Trinity College, Cambridge. Heinz Kurz and Neri Salvadori were kind enough to grant me permission to quote from their unpublished papers.

## Bibliography

Albani, P., 1998, 'Sraffa and Wittgenstein. Profile of an Intellectual Friendship', *History of Economic Ideas*, vol. 4, 151-173.

Arena, R., 1982, 'P. Sraffa, L. Wittgenstein et la notion de jeux de langage', in *Économies et sociétés. Cahiers de l'I.S.M.E.A.*, vol. 16, 329-354.

Barone, E., 1967, 'The Ministry of Production in the Collectivist State', in F. A. Hayek (ed.), *Collectivist Economic Planning*, New York, August M. Kelley, 245-290

Bharadwaj, K., and B. Schefold, 1990, *Essays on Piero Sraffa. Critical Perspectives on the Revival of Classical Theory*, London, Routledge.

Bidard, C., 1998, 'L'évolution méthodologique de la théorie post-sraffienne', *Revue d'économie politique*, vol. 108, 778-797.

Bishop, E., 1967, *Foundations of Constructive Analysis*, New York, McGraw-Hill; second edition: E. Bishop & D. Bridges, *Constructive Analysis*, Berlin, Springer, 1985.

Boltzmann, L., 1974, *Theoretical Physics and Philosophical Problems*, Dordrecht, D. Reidel,.

Braithwaite, R. B., 1953, *Scientific Explanation*, Cambridge, Cambridge University Press.

Bridges, D. & F. Richman, 1987, *Varieties of Constructive Mathematics*, Cambridge, Cambridge University Press.

Brody, A., 1970, *Proportions, Prices and Planning*, Amsterdam, North Holland.

Brouwer, L. E. J., 1975, *Collected Works*, A. Heyting (ed.), Amsterdam, North-Holland, 2 vols.

Burmeister, E., 1968, 'On a Theorem of Sraffa', *Economica*, vol. 35, 83-87.

Carnap, R., 1931, 'Die physikalische Sprache als Universalssprache der Wissenschaft', *Erkenntnis*, vol. 2, 432-465.

Carnap, R., 1967, *The Logical Structure of the World*, Berkeley & Los Angeles, University of California Press.

Carnap, R., 1995, *The Unity of Science*, reprint: Bristol, Thoemmes. (Translation of R. Carnap, 'Die physikalische Sprache als Universalssprache der Wissenschaft', *Erkenntnis*, vol. 2, 1931 432-465.)

Chakravarty, S., 1989, 'John von Neumann's Model of an Expanding Economy: an Essay in Interpretation', in M. Dore, S. Chakravarty & R. Goodwin (eds.), *John von Neumann and Modern Economics*, Oxford, Clarendon Press, 69-81.

Dantzig, G. B., 1956, 'Constructive Proof of the Min-Max Theorem', *Pacific Journal of Mathematics*, vol. 6.

Davis, J. B., 1988, 'Sraffa, Wittgenstein and Neo-Classical Economics', *Cambridge Journal of Economics*, vol. 12, 29-36.

Davis, J. B., 1993, 'Sraffa, Interdependence and Demand: the Gramscian Influence', *Review of Political Economy*, vol. 5, 22-39.

Davis, J. B., 2002, 'Gramsci, Sraffa, Wittgenstein: Philosophical Linkages', *European Journal for the History of Economic Thought*, vol. 9, 384-401.

Debreu, G., 1959, *Theory of Value. An Axiomatic Analysis of Economic Equilibrium*, New York. John Wiley & Sons.

Dobb, M., 1973, *Theories of Value and Distribution since Adam Smith*, Cambridge, Cambridge University Press.

Dore, M. H. I., 1989, 'The Legacy of John von Neumann', in M. Dore, S. Chakravarty & R. Goodwin (eds.), *John von Neumann and Modern Economics*, Oxford, Clarendon Press, 82-99.

Dostaler, G., 1982, 'Marx et Sraffa', *L'actualité économique*, vol. 58, 95-114.

Eatwell, J., M. Milgate & P. Newman (eds.), 1987, *The New Palgrave. A Dictionary of Economics*, London, MacMillan, 4 vols.

Eatwell, J. & C. Panico, 1987b, 'Sraffa, Piero', in Eatwell, Milgate & Newman (1987), vol. 4, 445-452.

Edwards, H. M., 1995, *Linear Algebra*, Boston, Birkhäuser.

Fann, K.T., 1969, *Wittgenstein's Conception of Philosophy*, Los Angeles, University of California Press.

Gandy, R., 1982, 'Limitations to Mathematical Knowledge', in D. van Dalen, D. Lascar & T. J. Smiley (eds.), *Logic Colloquium '80*, Amsterdam, North-Holland, 129-146.

Georgescu-Roegen, N., 1951, 'The Aggregate Linear Production Function and its Application to von Neumann's Economic Model', in T. C. Koopmans (ed.), *Activity Analysis of Production and Allocation*, New York, John Wiley, 98-115.

Goodwin, R. M., 1951, 'Iteration, Automatic Computers, and Economic Dynamics', *Metroeconomica*, vol. 3, 1-7.

Goodwin, R. M. & L. F. Punzo, 1987, *The Dynamics of a Capitalist Economy*, Oxford, Polity

Press.

Gramsci, A., 1971, *Selections from the Prison Notebooks of Antonio Gramsci*, G. Nowell-Smith & Q. Hoare (eds.), New York, International Publishers.

Hayek, F. von, 1952, *The Sensory Order. An Inquiry into the Foundations of Theoretical Psychology*, Chicago, University of Chicago Press.

Hertz, H., 1899, *Principles of Mechanics*, London, McMillan.

Hintikka M. B. & J. Hintikka, 1986, *Investigating Wittgenstein*, Oxford, Blackwell.

Jacquette, D., 1998, *Wittgenstein's Thought in Transition*, West Lafayette IN., Purdue University Press.

Kaldor, N., 1984, 'Obituary, Piero Sraffa', *The Cambridge Review*, vol. 105, 149-150.

Kaldor, N., 1985, 'Piero Sraffa (1898-1983)', *Proceedings of the British Academy*, vol. 71, 615-640.

Kakutani, S., 1941, 'A Generalization of Brouwer's Fixed Point Theorem', *Duke Mathematical Journal*, vol. 8, 457-459.

Kemeny, J. G., Morgenstern, O., & G. L. Thompson, 1956, 'A Generalisation of the von Neumann Model of an Expanding Economy', *Econometrica*, vol. 24, 115-135.

Kienzler, W., 1997, *Wittgensteins Wende zur seiner Spätphilosophie 1930-1932*, Frankfurt, Surhkamp.

Kurz, H., 1998, 'Le fonds Sraffa à Cambridge. Aperçu sur les écrits non publiés', *Revue d'économie politique*, vol. 108, 813-837.

Kurz, H., 2003, 'The Surplus Interpretation of the Classical Economists', in J. Biddle, J. Davis and W. Samuels (eds.), *The Blackwell Companion to the History of Economic Thought*, Oxford, Blackwell, 167-183.

Kurz, H. & N. Salvadori, 1993, 'Von Neumann's Growth Model and the 'Classical' Tradition', *European Journal of the History of Economic Thought*, vol. 1, 130-160.

Kurz, H. & N. Salvadori, 2001, 'Sraffa and the Mathematicians: Frank Ramsey and Alister Watson', in T. Cozzi & R. Marchionatti (eds.), *Piero Sraffa's Political Economy. A Centenary Estimate*, Routledge, London, 254-284.

Kurz, H. & N. Salvadori, 2004a, 'Von Neumann, the Classical Economists and Arrow-Debreu: Some Notes', *Acta Oeconomica*, vol. 54, 39-62.

Kurz, H. & N. Salvadori, 2004b, 'On the Collaboration between Sraffa and Besicovitch: The Cases of Fixed Capital and Non-Basics in Joint Production', in *Convegno internazionale Piero Sraffa (Roma, 11-12 febbraio 2003)*, Atti dei Convegni Lincei, vol. 200, Rome, Accademia Nazionale dei Lincei, 255-301.

Kurz, H. & N. Salvadori, (forthcoming), "'Man from the Moon," On Sraffa's Objectivism'. Available at <http://www-dse.ec.unipi.it/salvadori/>

Lenin, 1970, *Materialism and Empirio-Criticism*, New York, International Publishers.

Mach, E., 1959, *The Analysis of Sensations*, New York, Dover.

Malcolm, N., 1984, *Ludwig Wittgenstein. A Memoir*, Oxford, Oxford University Press.

Mancosu, P. & M. Marion, 2003, 'Wittgenstein's Constructivization of Euler's Proof of the Infinity of Prime Numbers', in F. Stadler (ed.), *The Vienna Circle and Logical Empiricism : Re-evaluation and Future Perspectives*, Dordrecht, Kluwer, 171-188.

Marion, M., 1995a, 'Wittgenstein and Finitism', *Synthese*, vol. 105, 141-176.

Marion, M., 1995b, 'Kronecker's "safe haven of real mathematics"', in M. Marion and R. S. Cohen (eds.), *Québec Studies in the Philosophy of Science*, Dordrecht, Kluwer, vol. 1, 187-213.

Marion, M., 1998, *Wittgenstein, Finitism, and the Foundations of Mathematics*, Oxford, Oxford University Press.

Marion, M., 2003, 'Wittgenstein and Brouwer', *Synthese*, vol. 137, 103-127.

Marion, M., 2004, *Ludwig Wittgenstein. Introduction au "Tractatus logico-philosophicus"*, Paris, Presses Universitaires de France.

Marion, M., (forthcoming I), 'Wittgenstein et le pragmatisme britannique', in F. Latraverse (ed.), *Peirce, Wittgenstein et le pragmatisme*, Paris, L'Harmattan.

Marion, M., (forthcoming II), 'Wittgenstein and Sraffa's *anthropologische Betrachtungsweise*', in J. Hintikka & E. de Pellegrin (eds.), *Memorial Volume in Honor of Georg Henrik von Wright*, Dordrecht, Kluwer.

Marshall, A., 1952, *Principles of Economics*, 8<sup>th</sup> ed., London, MacMillan.

McGuinness, B. F., 1982, 'Freud and Wittgenstein', in B. F. McGuinness (ed.), *Wittgenstein and his Times*, Oxford, Blackwell, 27-43.

McGuinness, B. F., 'Sraffa, Wittgenstein e i problemi del metodo logico' in R. Bellofiore (ed.), *Tra teoria economica e grande cultura europea: Piero Sraffa*, Milano, Franco Angeli, 85-92.

- Monk, R., 1990, *Ludwig Wittgenstein. The Duty of Genius*, London, Jonathan Cape.
- Nadeau, R., 2001, 'Sur l'antiphysicalisme de Hayek. Essai d'élucidation', *Revue de philosophie économique*, vol. 3, 2001, 67-112.
- Naldi, N., 2000, 'The Friendship between Piero Sraffa and Antonio Gramsci in the Years 1919-1927', *European Journal for the History of Economic Thought*, vol. 7, 79-114.
- Neurath, O., 1983, *Philosophical Papers 1913-1946*, Dordrecht, D. Reidel.
- Newman, P., 1987, 'Ramsey, Frank Plumpton', in Eatwell, Milgate & Newman (1987), vol. 4, 41-46.
- Pasinetti, L. L., 1981, *Structural Change and Economic Growth*, Cambridge, Cambridge University Press.
- Petty, W., 1986, *The Economic Writings of Sir William Petty*, C. H. Hull (ed.), vol. 1, Cambridge, Cambridge University Press.
- Potier, J.-P., 1991, *Piero Sraffa. Unorthodox Economist (1898-1983)*, London, Routledge.
- Punzo, L., 1986, 'La matematica di Sraffa', in R. Bellofiore (ed.), *Tra Teoria economica e grande cultura europea*, Milano, Franco Angeli, 141-167.
- Punzo, L. F., 1989, 'Von Neumann and Karl Menger's Mathematical Colloquium' in M. Dore, S. Chakravarty & R. Goodwin (eds.), *John von Neumann and Modern Economics*, Oxford, Clarendon Press, 28-65.
- Punzo, L. F., 1991, 'The School of Mathematical Formalism and the Viennese Circle of Mathematical Economists', *Journal of the History of Economic Thought*, vol. 13, 1-18.
- Punzo, L. F. & K. Velupillai, 1984, 'Multisectoral Models and Joint Production', in F. van der Ploeg (ed.), *Mathematical Methods in Economics*, New York, John Wiley & Sons, 57-92.
- Rhees, R., 1996, *Discussions of Wittgenstein*, reprint: Bristol, Thoemmes.
- Robinson, J., 1965, 'Piero Sraffa and the Rate of Exploitation', *The New Left Review*, n. 35, 28-34.
- Roncaglia, A., 1978, *Sraffa and the Theory of Prices*, New York, John Wiley & Sons.
- Roncaglia, A., 1983, 'Piero Sraffa and the Reconstruction of Political Economy', *Banca Nazionale del Lavoro Quarterly Review*, vol. 36, 337-350.
- Roncaglia, A., 2000, *Piero Sraffa. His Life, Thought and Cultural Heritage*, London, Routledge.

Samuelson, P. (1990). 'Revisionist Findings on Sraffa', in Bharadwaj & Schefold (1990), 263-301.

Salvadori, N., 1984, 'Le choix des techniques chez Sraffa: le cas de la production jointe', in C. Bidard (ed.), *La Production jointe. Nouveaux débats*, Paris, Economica, 175-185.

Salvadori, N., 1985, 'Was Sraffa making no Assumptions on Returns?', *Metroeconomica*, vol. 36, 175-186.

Sen, A., 2003, 'Sraffa, Wittgenstein, and Gramsci', *Journal of Economic Literature*, vol. 41, 1240-1255.

Sen, A., 2004, 'Piero Sraffa: A Student's Perspective', in *Convegno internazionale Piero Sraffa (Roma, 11-12 febbraio 2003)*, Atti dei Convegni Lincei, vol. 200, Rome, Accademia Nazionale dei Lincei, 23-60.

Schefold, B., 1978, 'On Counting Equations', *Zeitschrift für Nationalökonomie*, vol. 38, 253-285.

Schefold, B., 1980, 'Von Neumann and Sraffa: Mathematical Equivalence and Conceptual Difference', *Economic Journal*, vol. 90, 140-156.

Schefold, B., 2004, 'Joint Production: Triumph of Economic over Mathematical Logic?', in *Convegno internazionale Piero Sraffa (Roma, 11-12 febbraio 2003)*, Atti dei Convegni Lincei, vol. 200, Rome, Accademia Nazionale dei Lincei, 303-331.

Schmidt, C., 1985, *La sémantique économique en question*, Paris, Calmann-Levy.

Schmidt, C., 1988, 'Le problème des fondements en théorie économique en relation avec le problème des fondements en mathématiques', *Cahiers d'épistémologie* no. 8805, Département de philosophie, Université du Québec à Montréal.

Schmidt, C., 1990, 'Les fondements d'une critique constructiviste de l'économie marginaliste', in *Arena* (1990), 99-121.

Sraffa, P., 1926, 'The Laws of Returns Under Competitive Conditions', *The Economic Journal*, vol. 36, 535-550; reprinted in G. Stigler & K. Boulding (eds.), *Readings in Price Theory*, Chicago, Richard D. Irwin, 180-197.

Sraffa, P., 1951, 'Introduction to Ricardo's *On the Principles of Political Economy and Taxation*', in Ricardo (1951-73), vol. 1, xiii-lxii.

Sraffa, P., 1960, *Production of Commodities by Means of Commodities. Prelude to a Critique of Economic Theory*, Cambridge, Cambridge University Press.

Steedman, I., 1976, 'Positive Profits with Negative Surplus Value: A Reply to Wolfstetter', *Economic Journal*, vol. 86, 873-876.

Stegmüller, W., 1969, *Main Currents in German, British and American Philosophy*, Bloomington, Indiana University Press

Troelstra, A. & D. van Dalen, 1988, *Constructivism in Mathematics. An Introduction*, Amsterdam, North-Holland, 2 vols.

Vellupillai, K., 2000, *Computable Economics*, Oxford University Press.

Ville, J., 1938, 'Sur la théorie générale des jeux', dans E. Borel, *Traité du calcul des probabilités*, vol. 4, Paris, Gauthier-Villars, 105-113.

von Neumann, J., 1928, 'Zur Theorie der Gesellschaftsspiele', *Mathematische Annalen*, vol. 100, 295-320.

von Neumann, J., 1937, 'Über ein Ökonomisches Gleichungssystem und eine Verallgemeinerung des Brouwerschen Fixpunktsatzes', *Ergebnisse eines Mathematischen Kolloquiums*, vol. 8, 73-83.

von Neumann, J., 1945, 'A Model of General Economic Equilibrium', *Review of Economic Studies*, vol. 13, 1-9. (Translation of von Neumann 1937.)

Watson, A. G. D., 1938, 'Mathematics and its Foundations', *Mind*, vol. 47, 440-451.

Whitehead, A. N., 1925, *Science and the Modern World*, New York, MacMillan.

Wittgenstein, L., 1922, *Tractatus Logico-Philosophicus*, London, Routledge & Kegan Paul.

Wittgenstein, L., 1953, *Philosophical Investigations*, Oxford, Blackwell.

Wittgenstein, L., 1958, *The Blue and Brown Books*, Oxford, Blackwell.

Wittgenstein, L., 1974, *Philosophical Grammar*, Oxford, Blackwell

Wittgenstein, L., 1975, *Philosophical Remarks*, Oxford, Blackwell.

Wittgenstein, L., 1976, *Wittgenstein's Lectures on the Foundations of Mathematics, Cambridge 1939*, Ithaca, Cornell University Press.

Wittgenstein, L., 1978, *Remarks on the Foundations of Mathematics*, third edition, Oxford, Blackwell.

Wittgenstein, L., 1979, *Ludwig Wittgenstein and the Vienna Circle*, Oxford, Blackwell.

Wittgenstein, L., 1980, *Culture and Value*, Oxford, Blackwell.

Wittgenstein, L., 1994, *Wiener Ausgabe*, vol. 2, Vienna, Springer.

Wittgenstein, L., 1996, *Wiener Ausgabe*, vol. 5, Vienna, Springer.

Wittgenstein, L., 1995, *Cambridge Letters*, Oxford, Blackwell.



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